The background is a solid purple color. On the left side, there are decorative elements: a light purple semi-circle at the top, a white zigzag line, and a light purple semi-circle below it. A yellow circle is positioned on the left side of the white border. On the bottom right, there is a yellow semi-circle.

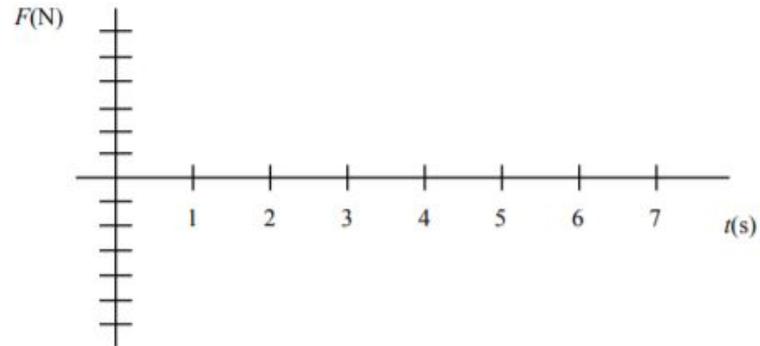
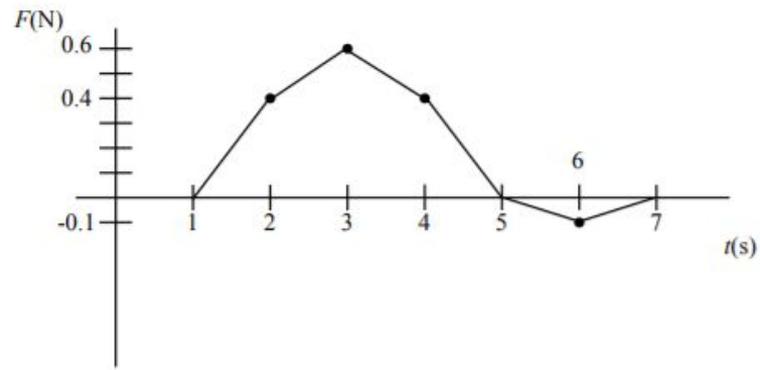
Week 11 Tutorial

Tutorial Section

Tutorial Time

Tutorial TA Name

Question 1



Two carts collide on a frictionless track aligned with the x-axis. The force of cart 1 on cart 2 versus time graph is shown in the top figure.

- Graph the force of cart 2 on cart 1 versus time on the bottom figure. Label the lower graph $F_{2\text{ on }1}$ and put numbers on the vertical axis. /4
- Find the impulse that cart 1 puts on cart 2. /3

Question 1

- (c) If cart 2 has a mass of 500 g and an initial velocity $v_{ix} = -1$ m/s, find cart 2's velocity following the collision. /4
- (d) Assuming that cart 1 starts to the left of cart 2 (and the positive x-axis is to the right), label the regions of the force vs. time graph as having a repulsive or an attractive force. /2
- (e) What could cause an attractive force between the carts? (We have four end types on our carts: magnets, springs, clay or Velcro. /2
- (f) If the collision was perfectly inelastic, at what final speed would cart 1 end up? /3
- (g) If cart 1 had a mass of 1.5 kg and underwent a perfectly inelastic collision what was its initial speed? /3

Problem Solving Framework

PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH **16**, 010123 (2020)

Template for teaching and assessment of problem solving in introductory physics

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A. V. Fritz ⁴ and C. E. Wieman ^{1,3}

1. Framing

Visual Representation

Assumptions and Simplifications

Relevant Concepts

Information Needed

Similar Problems

2. Planning

Solution Plan

Rough Estimate

3. Execution

Carry-out Plan for solving

- Work in algebra/symbols until the BITTER end
- Plug in numbers at the LAST step

4. Answer Checking

Compare to Estimate

Units Check

Limits Test

Getting (UnStuck)

1. Framing

2. Planning

3. Execution

4. Answer Checking

1. Framing



Visual representation: Consider how the carts will be moving before and after collision.



Assumptions and simplifications: Only consider this motion in 1D. Friction is negligible.



Relevant concept: Collisions, linear momentum, and impulse.



Information needed: Mass of carts, initial velocities of carts, area under force vs time graph.



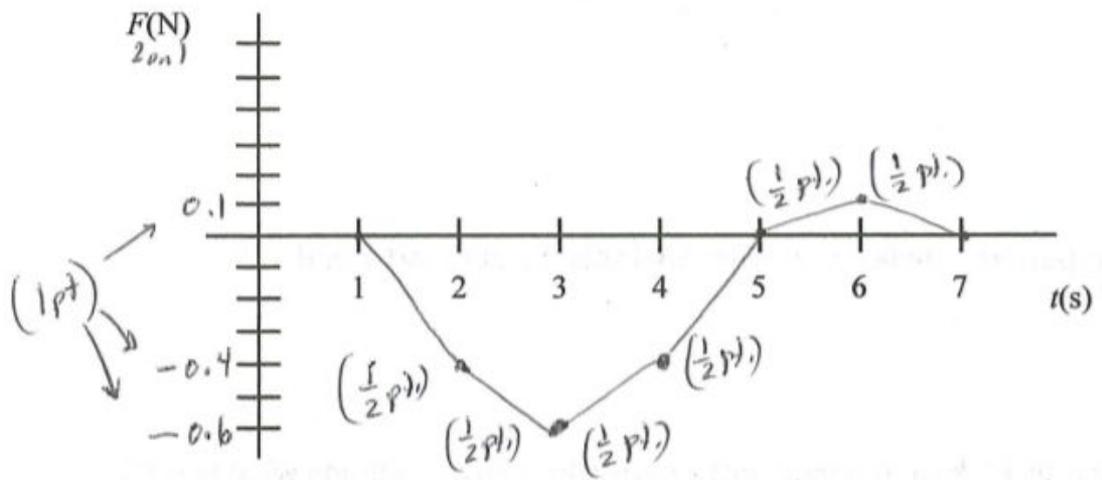
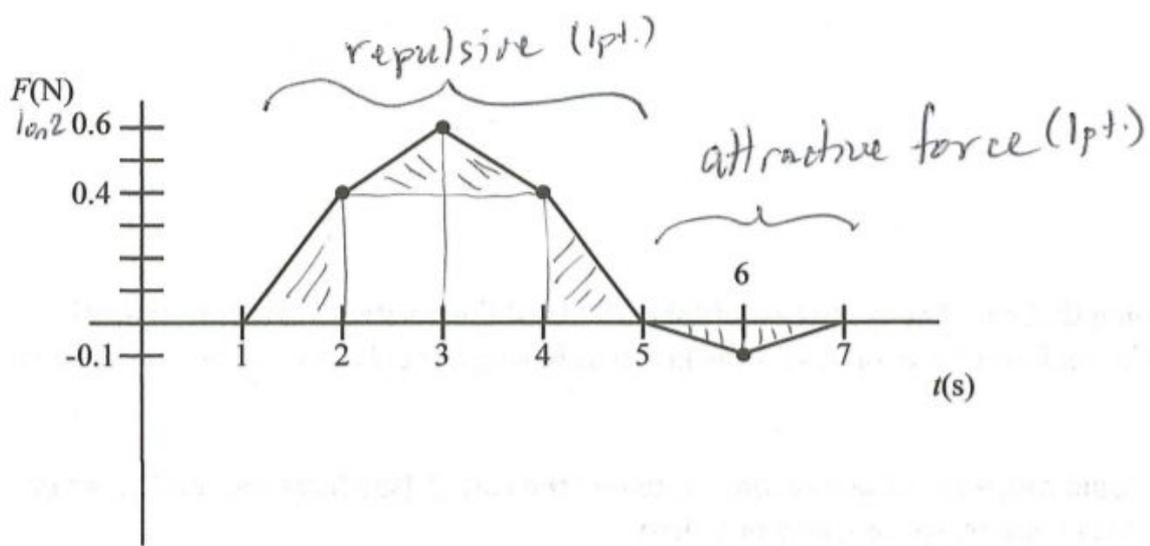
Similar problems: Problems involving collisions between two bodies.

2. Planning

- Rough estimate: Estimate the area under the force versus time graph. Estimate the velocity of the carts after collision.
- Solution plan: Use the initial velocity of the carts and impulse to solve for the final velocity of the carts.

3. Execution

See Slides 10-13



Two carts collide on a frictionless track aligned with the x-axis. The force of cart 1 on cart 2 versus time graph is shown in the top figure.

(a) Graph the force of cart 2 on cart 1 versus time on the bottom figure. /4

(b) Find the impulse that cart 1 puts on cart 2. /3

$$J_x = \text{area under } F_{1 \text{ on } 2} \text{ vs time}$$

$$= \text{area} \left(\begin{array}{c} \boxed{\text{diagonal lines}} \quad 0.4 \text{ N} \\ \text{1 s} \end{array} \quad \begin{array}{c} \boxed{\text{diagonal lines}} \quad 0.2 \text{ N} \\ \text{1 s} \end{array} \quad \begin{array}{c} \boxed{\text{empty}} \quad 0.4 \text{ N} \\ \text{2 s} \end{array} \quad \begin{array}{c} \boxed{\text{vertical lines}} \quad -0.1 \text{ N} \\ \text{1 s} \end{array} \right)$$

show shapes here or on the graph (1 pt.)

$$= (0.4 \text{ N}\cdot\text{s} + 0.2 \text{ N}\cdot\text{s} + 0.8 \text{ N}\cdot\text{s} - 0.1 \text{ N}\cdot\text{s}) \quad (1 \text{ pt.})$$

$$= (1.4 - 0.1) \text{ N}\cdot\text{s} = 1.3 \text{ N}\cdot\text{s} = 1.3 \text{ kg}\cdot\frac{\text{m}}{\text{s}} \quad (1 \text{ pt.})$$

- (c) If cart 2 has a mass of 500 g and an initial velocity $v_{ix} = -1$ m/s, find cart 2's velocity following the collision.

$$J_x = 1.3 \text{ kg} \cdot \frac{\text{m}}{\text{s}} = \Delta p_{2x} = m_2 (v_{2fx} - v_{2ix})$$

either \rightarrow $2.6 \text{ m/s} = v_{2fx} + 1 \text{ m/s}$ $v_{2fx} = 1.6 \text{ m/s}$

- (d) Assuming that cart 1 starts to the left of cart 2 (and the positive x-axis is to the right), label the regions of the force vs. time graph as having a repulsive or an attractive force.

- (e) What could cause an attractive force between the carts? (We have four end types on our carts: magnets, springs, clay or Velcro.)

Either clay or velcro would allow the carts to try to stick together following the collision, as they tried to separate both would tend to pull them back together.

(f) If the collision was perfectly inelastic, at what final speed would cart 1 end up? 13

$v_{1fx} = v_{2fx} = 1.6 \text{ m/s}$ if the collision was perfectly inelastic as the objects would stick together and have to travel at the same speed. (1pt)

(g) If cart 1 had a mass of 1.5 kg and underwent a perfectly inelastic collision what was its initial speed? 13

$$J_{on}x = -1.3 \text{ kg} \cdot \frac{\text{m}}{\text{s}} = \Delta p_{ix} = m_1 (v_{1fx} - v_{1ix})$$
$$= 1.5 \text{ kg} (1.6 \text{ m/s} - v_{1ix}) \leftarrow \text{either}$$

$$-0.86 \text{ m/s} = 1.6 \text{ m/s} - v_{1ix}$$

$$v_{1ix} = 2.47 \text{ m/s} \quad (1pt)$$

Assess: Cart 1's mass is three times Cart 2's mass, so it's not surprising that the final speed of both carts is three times closer to the initial velocity of cart 1 than to the initial velocity of cart 2.

$$\left| \frac{\Delta v_{x1}}{\Delta v_{x2}} \right| = \frac{1}{3} = \frac{m_2}{m_1} \left\{ \begin{array}{l} \text{alternate} \\ \text{way} \\ (2 \text{ pts instead}) \end{array} \right.$$
$$|\Delta p_{1x}| = |\Delta p_{2x}|$$

4. Answer Checking

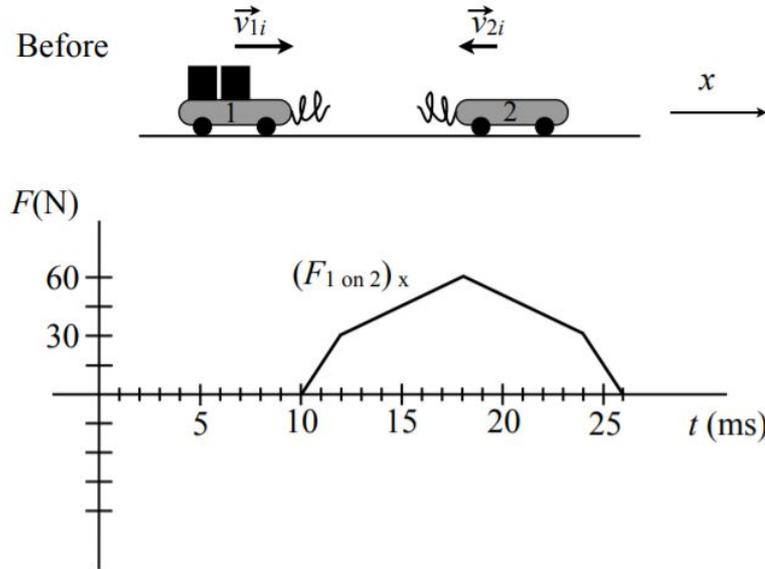
Compare to estimates: Check to see if the magnitude and sign of the impulse and final velocities of the carts match your estimates.

Units: Think about what units we use for impulse and velocity. Make sure they match your result.

Limits: Think about what would be some realistic values for the velocities of the carts you calculated. Do your answers make sense? Do they seem WAY far off?

Getting (UnStuck)? If you get stuck, go to the next question and come back to this one later.

Question 2



1. Cart 1 of mass 1.500 kg collides with cart 2 of mass 0.500 kg. A pictorial representation of the two carts just before the collision is shown above a graph of the x -component of the force exerted by cart 1 on cart 2 during the collision as a function of time.

(a) On Fig. 1 draw the x -component of the force exerted by cart 2 on cart 1 as a function of time. /6

(b) Calculate the x -component of the impulse of cart 1 on cart 2 during this collision. /4

Question 2

(c) What is the x -component of the impulse of cart 2 on cart 1 during this collision? Explain your result. /2

(d) By how much does the x -component of the momentum of cart 2 change during this collision? /2

(e) By how much does the x -component of the momentum of cart 1 change during this collision? /2

(f) What is the change in the x -component of the velocity of cart 2 during this collision? /3

(g) What is the change in the x -component of the velocity of cart 1 during this collision? /3

1. Framing

2. Planning

3. Execution

4. Answer Checking

1. Framing



Visual representation: Consider how the carts will be moving before and after collision.



Assumptions and simplifications: Only consider this motion in 1D. Friction is negligible.



Relevant concept: Collisions, linear momentum, and impulse.



Information needed: Mass of carts, area under force vs time graph.



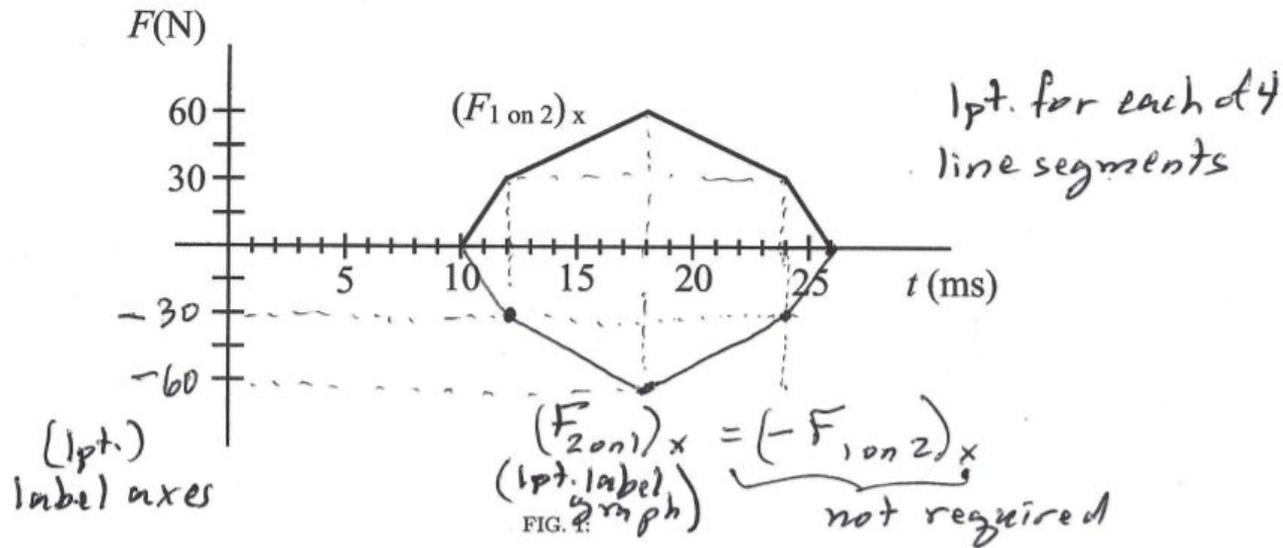
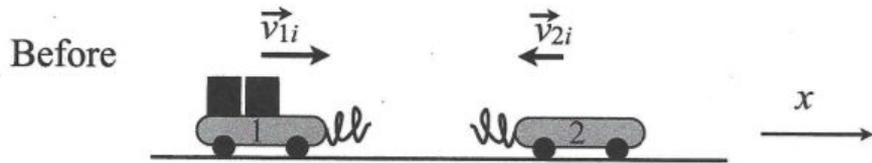
Similar problems: Problems involving collisions between two bodies.

2. Planning

- Rough estimate: Estimate the area under the force versus time graph. Estimate the velocity of the carts after collision.
- Solution plan: Use the area under the force vs time graph and the mass of the carts to solve for the impulse and the change in velocities of the carts.

3. Execution

- See slides 21-23.



1. Cart 1 of mass 1.500 kg collides with cart 2 of mass 0.500 kg. A pictorial representation of the two carts just before the collision is shown above a graph of the x -component of the force exerted by cart 1 on cart 2 during the collision as a function of time.

(a) On Fig. 1 draw the x -component of the force exerted by cart 2 on cart 1 as a function of time. /6

(b) Calculate the impulse of cart 1 on cart 2 during this collision.

14

$$\begin{aligned} J_{x1on2} &= \text{area under } (F_{1on2})_x \text{ vs time graph} \\ &= \underbrace{2 \times \frac{1}{2} (30\text{N}) (2 \times 10^{-3}\text{s})}_{\text{area of two triangles (1pt.)}} + \underbrace{\frac{1}{2} (30\text{N}) (12 \times 10^{-3}\text{s})}_{\text{area of two central triangles (1pt.)}} + \underbrace{(30\text{N}) (12 \times 10^{-3}\text{s})}_{\text{area of rectangle (1pt.)}} \\ &= 0.6 \text{ N}\cdot\text{s} \text{ (1pt.)} \end{aligned}$$

(c) What is the impulse of cart 2 on cart 1 during this collision? Explain your result.

12

$$\begin{aligned} J_{x2on1} &= \text{area under } (F_{2on1})_x \text{ vs time graph} \\ &= -J_{x1on2} \leftarrow \begin{matrix} \text{(1pt)} \\ \text{(1pt)} \end{matrix} \rightarrow \text{(Newton's third law)} \\ &= -0.6 \text{ N}\cdot\text{s} \text{ (1pt.)} \end{aligned}$$

(-1/2 if incorrect sign)

(d) By how much does the momentum of cart 2 change during this collision?

12

$$\begin{aligned} \Delta P_{2x} &= J_{x1on2} = 0.6 \text{ N}\cdot\text{s} = 0.6 \text{ kg} \frac{\text{m}}{\text{s}} \\ &\text{(1pt.)} \qquad \qquad \qquad \text{(1pt.)} \end{aligned}$$

(e) By how much does the momentum of cart 1 change during this collision?

1/2

$$\Delta P_{1x} = J_{x \text{ on } 1} = -0.6 \text{ N}\cdot\text{s} = -0.6 \text{ kg}\frac{\text{m}}{\text{s}}$$

(1pt.) (1pt.)

(-1/2 if incorrect sign)

(f) What is the change in velocity of cart 2 during this collision?

1/3

$$\Delta v_{2x} = \frac{\Delta P_{2x}}{m_2} = \frac{0.6 \text{ kg}\frac{\text{m}}{\text{s}}}{0.5 \text{ kg}} = 1.2 \text{ m/s}$$

(1pt.) (1pt.) (1pt.)

(g) What is the change in velocity of cart 1 during this collision?

1/3

$$\Delta v_{1x} = \frac{\Delta P_{1x}}{m_1} = -\frac{0.6 \text{ kg}\frac{\text{m}}{\text{s}}}{1.5 \text{ kg}} = -0.4 \text{ m/s}$$

(1pt.) (1pt.)

4. Answer Checking

Compare to estimates: Check to see if the magnitude and sign of the impulse and change in velocities of the carts match your estimates.

Units: Think about what units we use for impulse and velocity. Make sure they match your result.

Limits: Think about what would be some realistic values for the change in the velocities of the carts you calculated. Do your answers make sense? Do they seem WAY far off?

Getting (UnStuck)? If you get stuck, go to the next question and come back to this one later.