Physics 111 - Class 12A
Rotational Motion

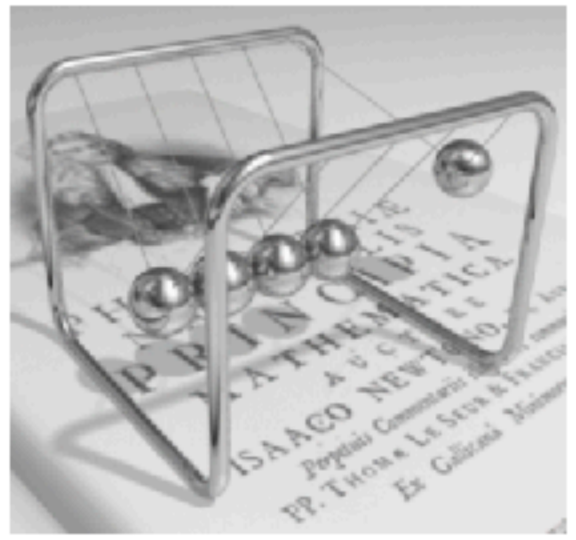
November 21, 2022

Class Outline

- Logistics / Announcements
- Chapter 10 Section Summary - Rotational Motion
    - Lots of talking from me today, SORRY!

Logistics/Announcements

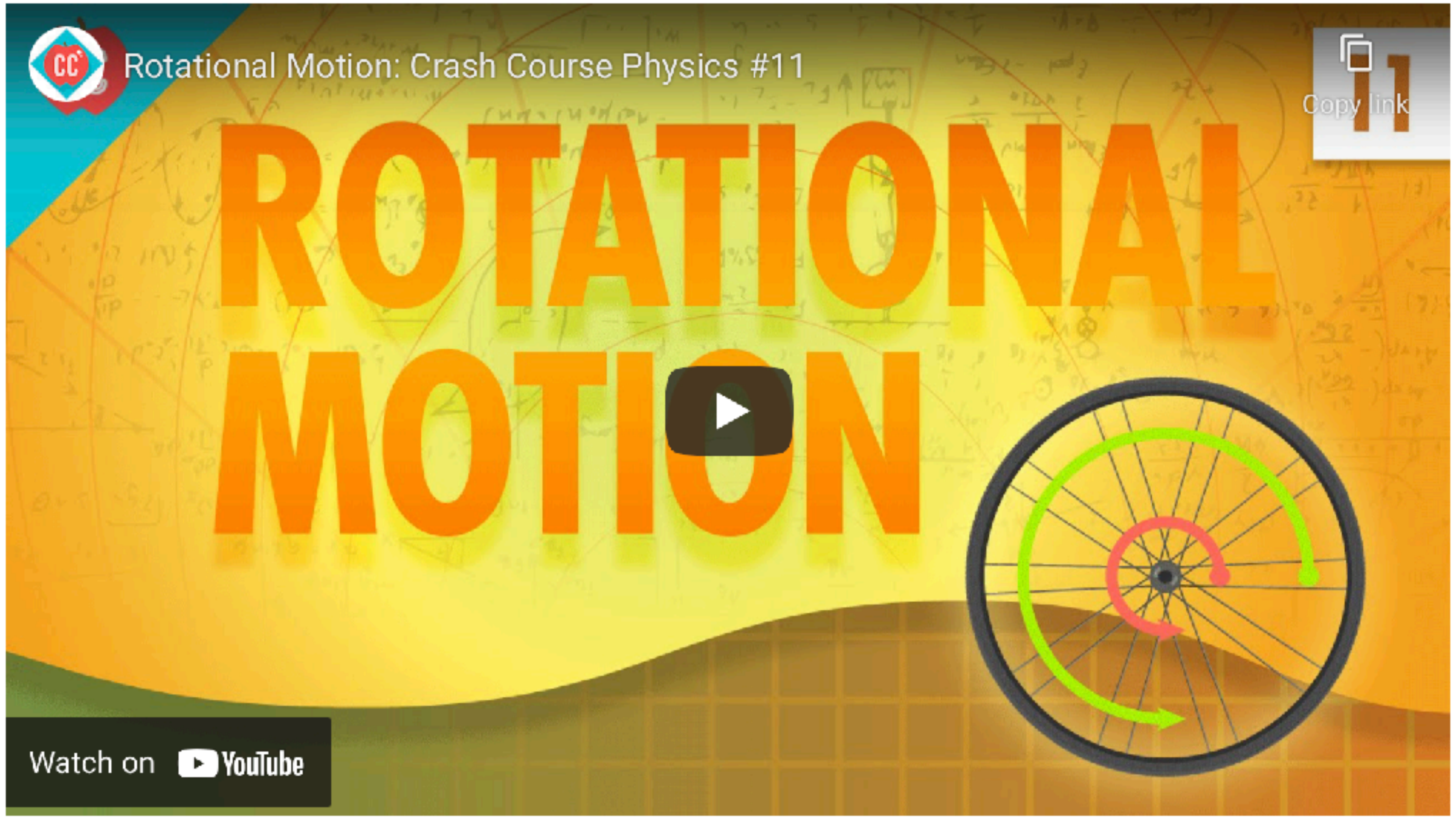
- Lab this week: Lab 8
- HW10 due this week on Thursday at 6 PM
- Learning Log 10 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Bonus Test 5 available this week (Chapters 8 & 9)
    - Test will be **in class on Friday from 4 - 5 PM**



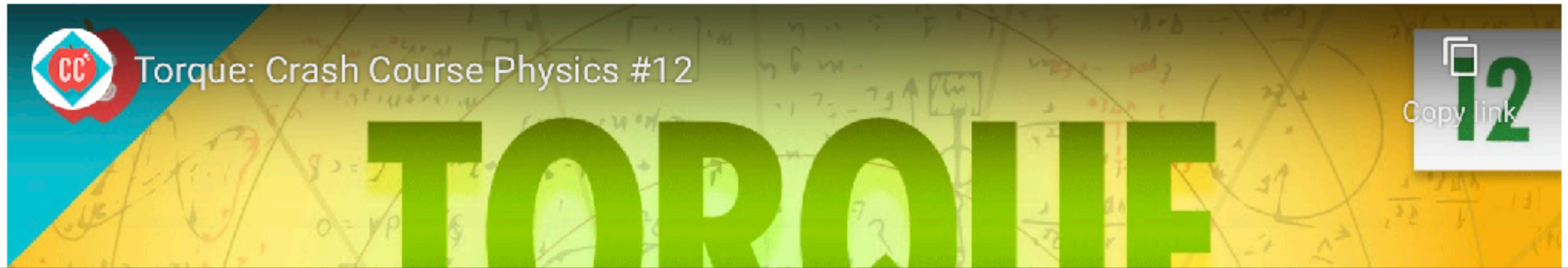
Physics 111

Search this book...

Rotational Motion



Torque



Monday’s Class

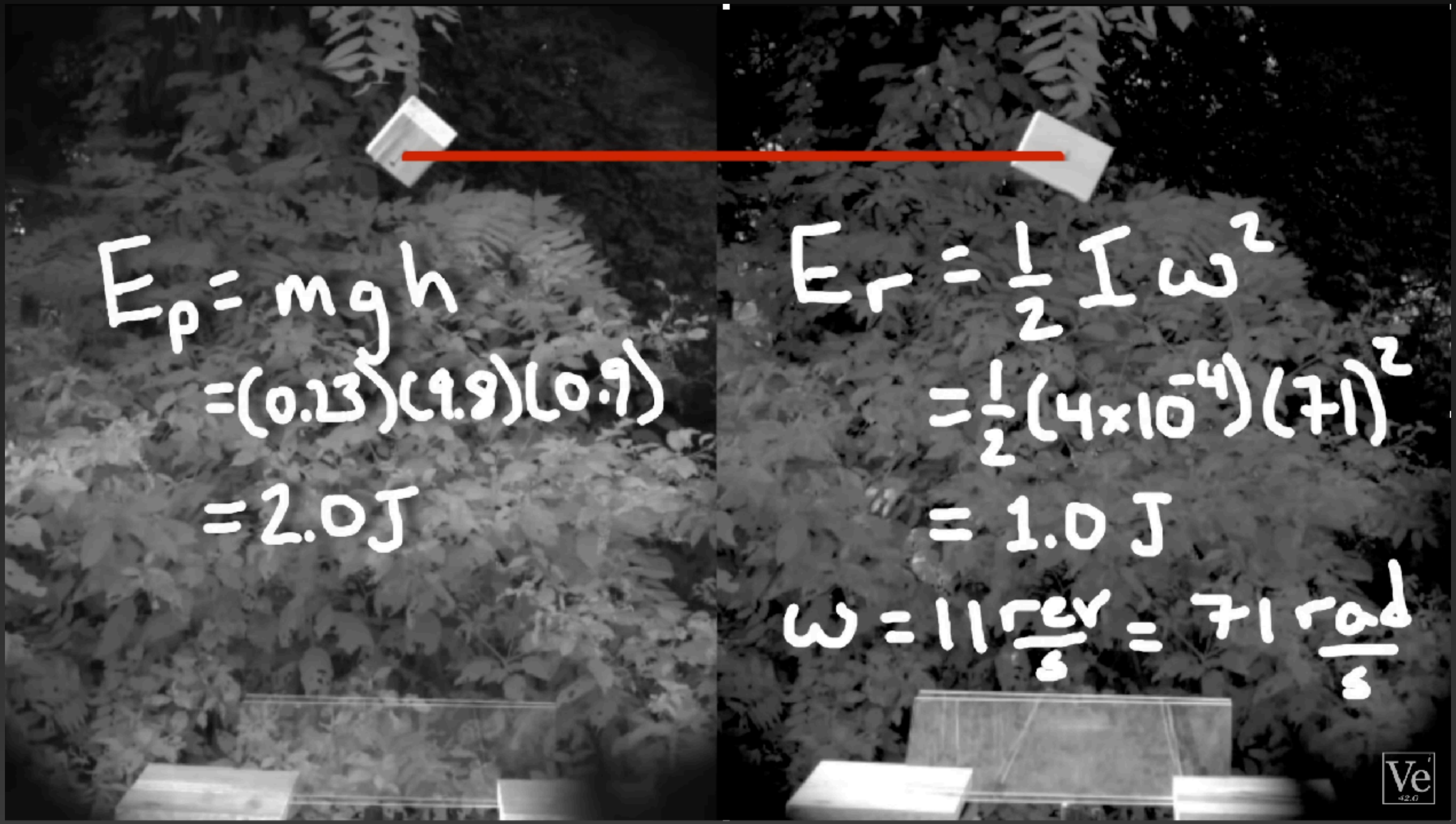
10.1 Rotational Variables
10.2 Rotation with Constant Angular Acceleration
10.3 Relating Angular and Translational Quantities
10.8 Work and Power for Rotational Motion

Rotational Variables

- So far in this course we have mostly done "translational motion" in x, y, or z
  - Quantities: Displacement, velocity, and acceleration
- As we become more sophisticated physicists, we realize that we have ignored "**rotational motion**"
  - Quantities: Angular displacement, Angular velocity, Angular acceleration

Rotational Variables

- Remember from the demo last week:
  - Spinning Block has rotational kinetic Energy



Rotational Quantities

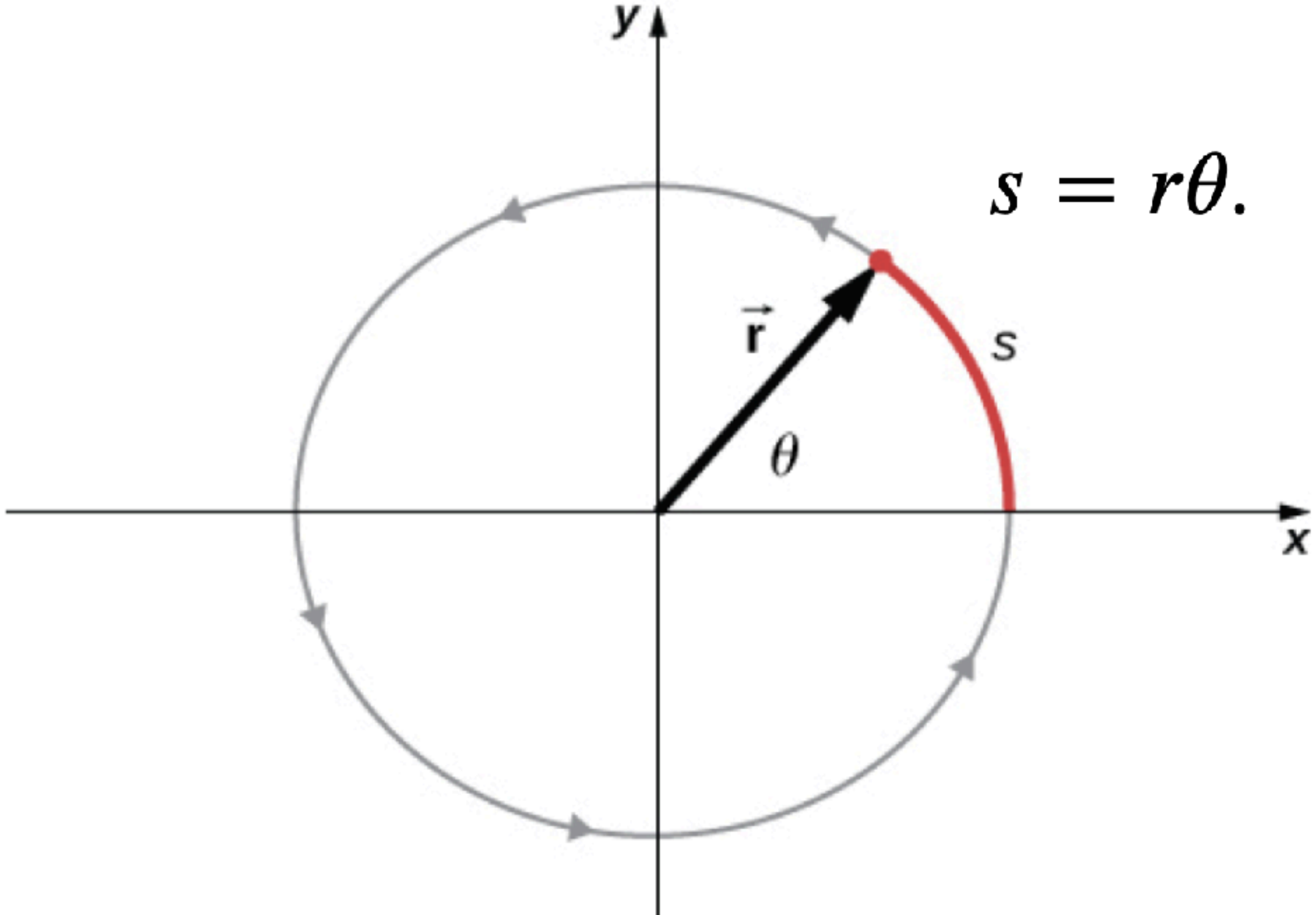
Rotational	Translational	Relationship (r = radius)
θ	s	$\theta = \frac{s}{r}$
ω	v_t	$\omega = \frac{v_t}{r}$
α	a_t	$\alpha = \frac{a_t}{r}$
	a_c	$a_c = \frac{v_t^2}{r}$

Table 10.3 Rotational and Translational Quantities: Circular Motion

Rotational Motion

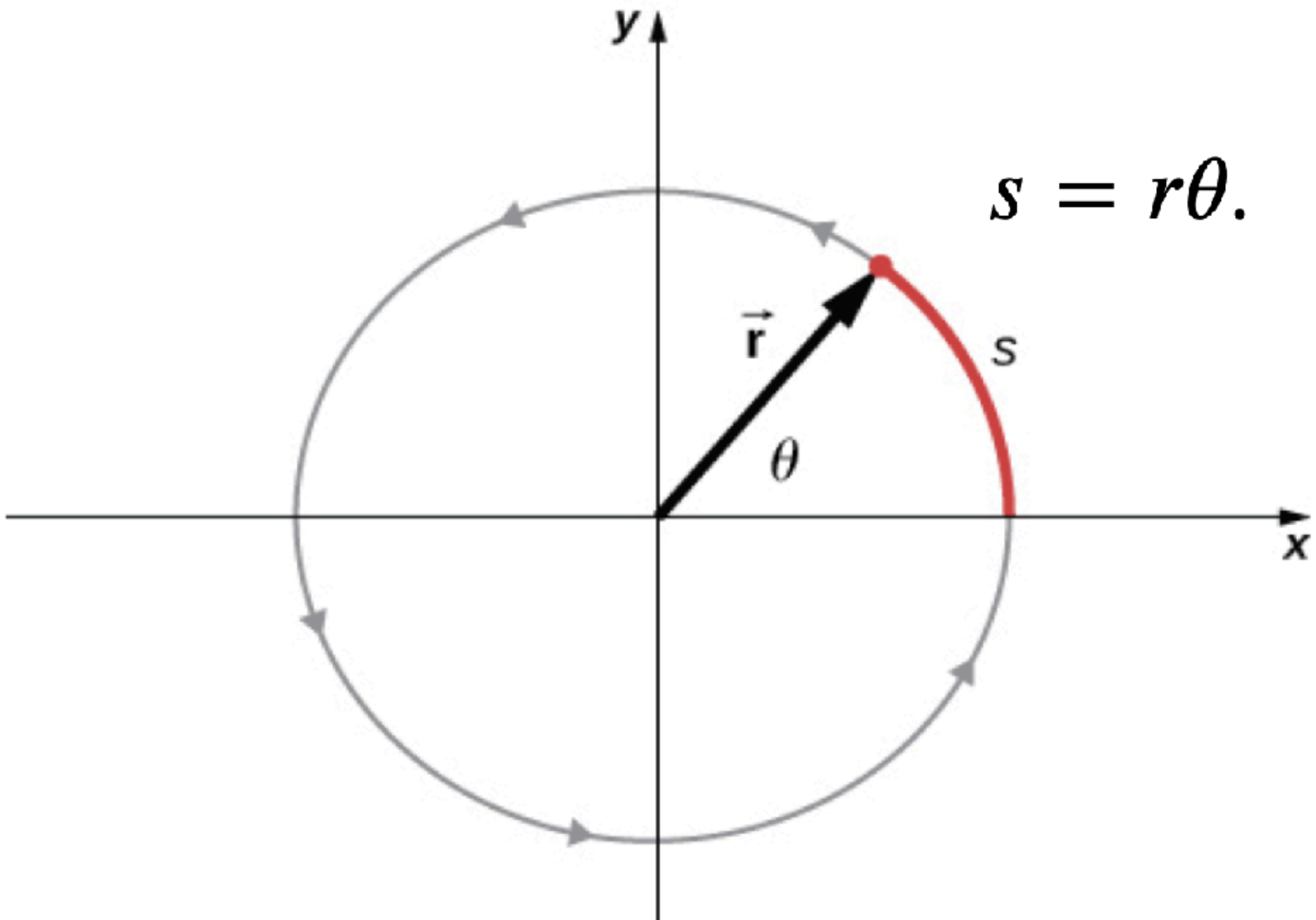
In Figure 10.2, we show a particle moving in a circle. The coordinate system is fixed and serves as a frame of reference to define the particle's position. Its position vector from the origin of the circle to the particle sweeps out the angle θ, which increases in the counterclockwise direction as the particle moves along its circular path. The angle θ is called the **angular position** of the particle. As the particle moves in its circular path, it also traces an arc length s.

$$s = r\theta.$$



Rotational Motion

In Figure 10.2, we show a particle moving in a circle. The coordinate system is fixed and serves as a frame of reference to define the particle's position. Its position vector from the origin of the circle to the particle sweeps out the angle θ, which increases in the counterclockwise direction as the particle moves along its circular path. The angle θ is called the **angular position** of the particle. As the particle moves in its circular path, it also traces an arc length s.

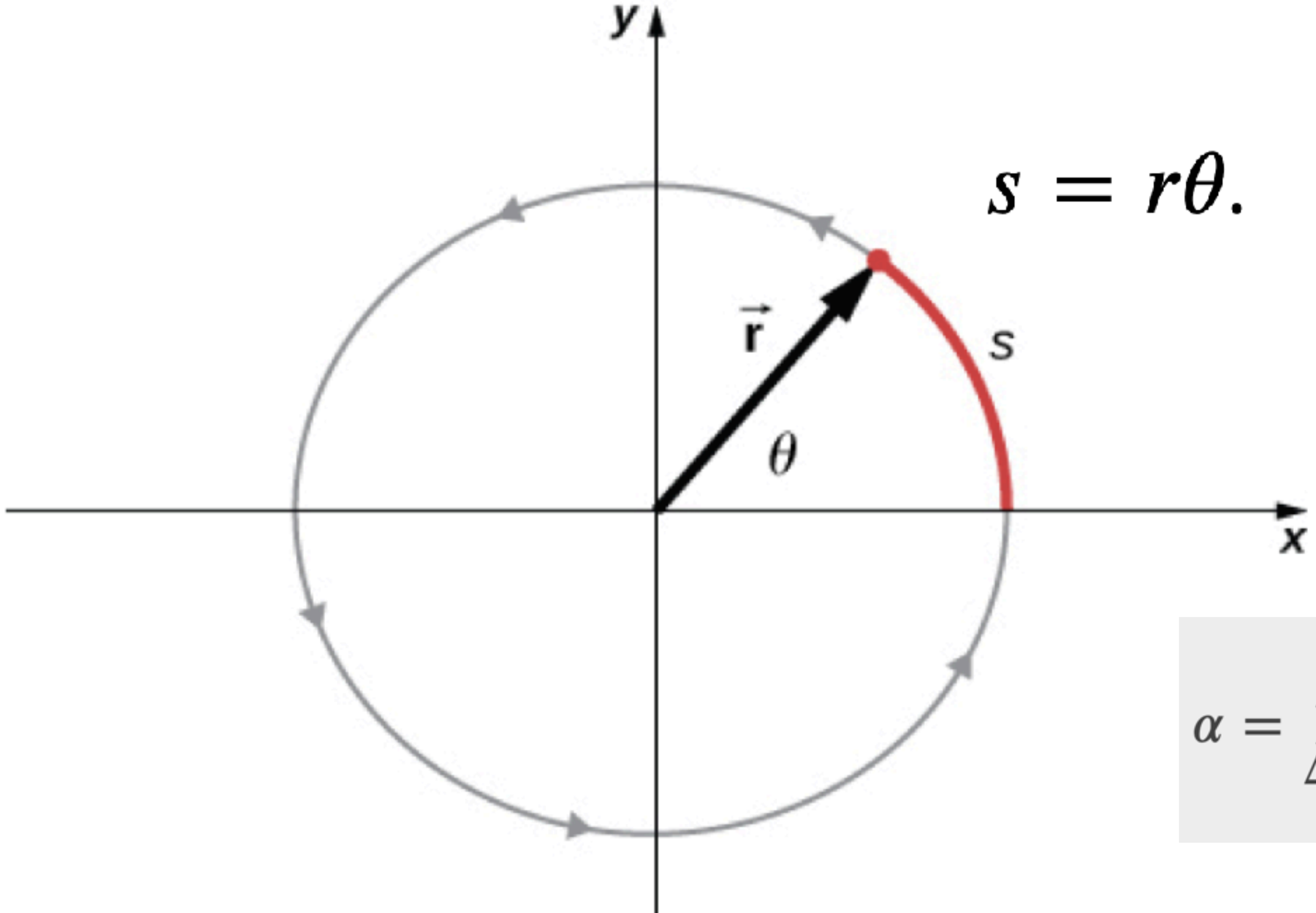


$$s = r\theta.$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt},$$

Rotational Motion

In Figure 10.2, we show a particle moving in a circle. The coordinate system is fixed and serves as a frame of reference to define the particle's position. Its position vector from the origin of the circle to the particle sweeps out the angle θ, which increases in the counterclockwise direction as the particle moves along its circular path. The angle θ is called the **angular position** of the particle. As the particle moves in its circular path, it also traces an arc length *s*.



$$s = r\theta.$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt},$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2},$$

Rotating Disk

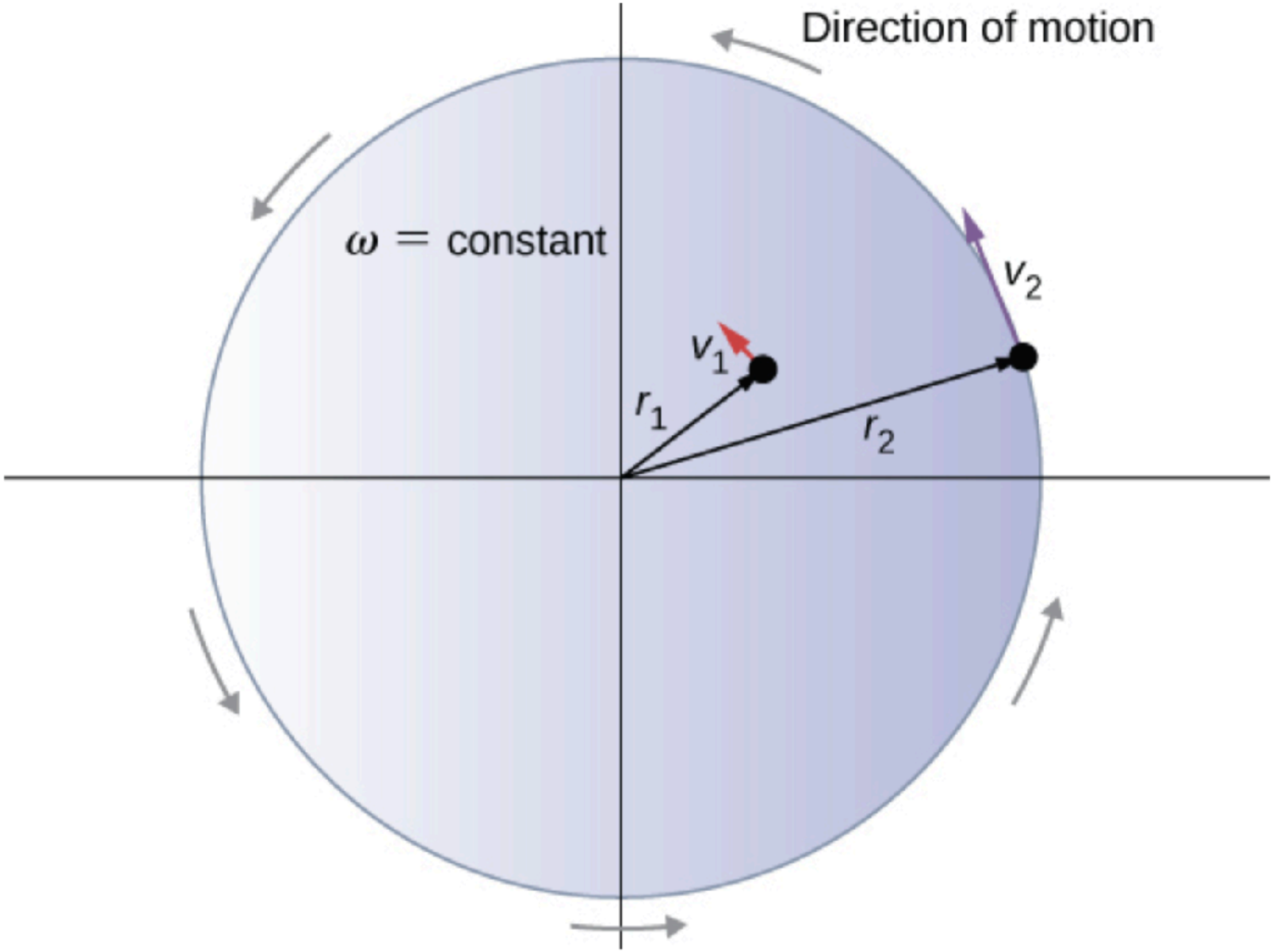
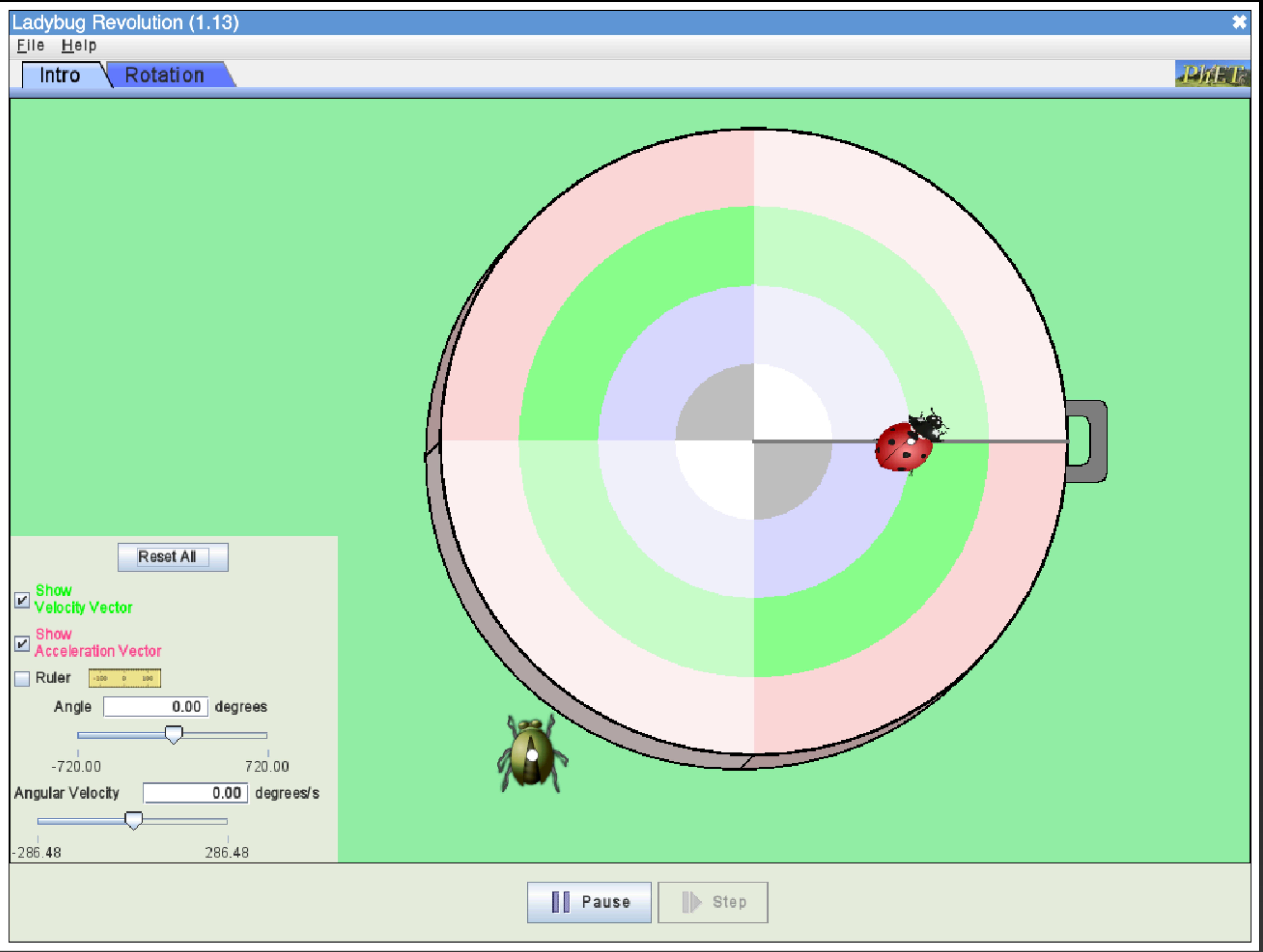


Figure 10.4 Two particles on a rotating disk have different tangential speeds, depending on their distance to the axis of rotation.

Rotating Disk



Link to PHET Simulation: https://phet.colorado.edu/sims/cheerpj/rotation/latest/rotation.html?simulation=rotation

Rotational Analogues

Translational
$x = x_0 + \bar{v}t$
$v_f = v_0 + at$
$x_f = x_0 + v_0 t + \frac{1}{2}at^2$
$v_f^2 = v_0^2 + 2a(\Delta x)$

Table 10.2 Rotational and Translational Kinematic Equations

Rotational Analogues

Rotational	Translational
$\theta_f = \theta_0 + \bar{\omega} t$	$x = x_0 + \bar{v} t$
$\omega_f = \omega_0 + \alpha t$	$v_f = v_0 + at$
$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x_f = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$	$v_f^2 = v_0^2 + 2a(\Delta x)$

Table 10.2 Rotational and Translational Kinematic Equations

Rotational Analogues

Translational
m
$K = \frac{1}{2}mv^2$

Table 10.4 Rotational and Translational Kinetic Energies and Inertia

Rotational Analogues

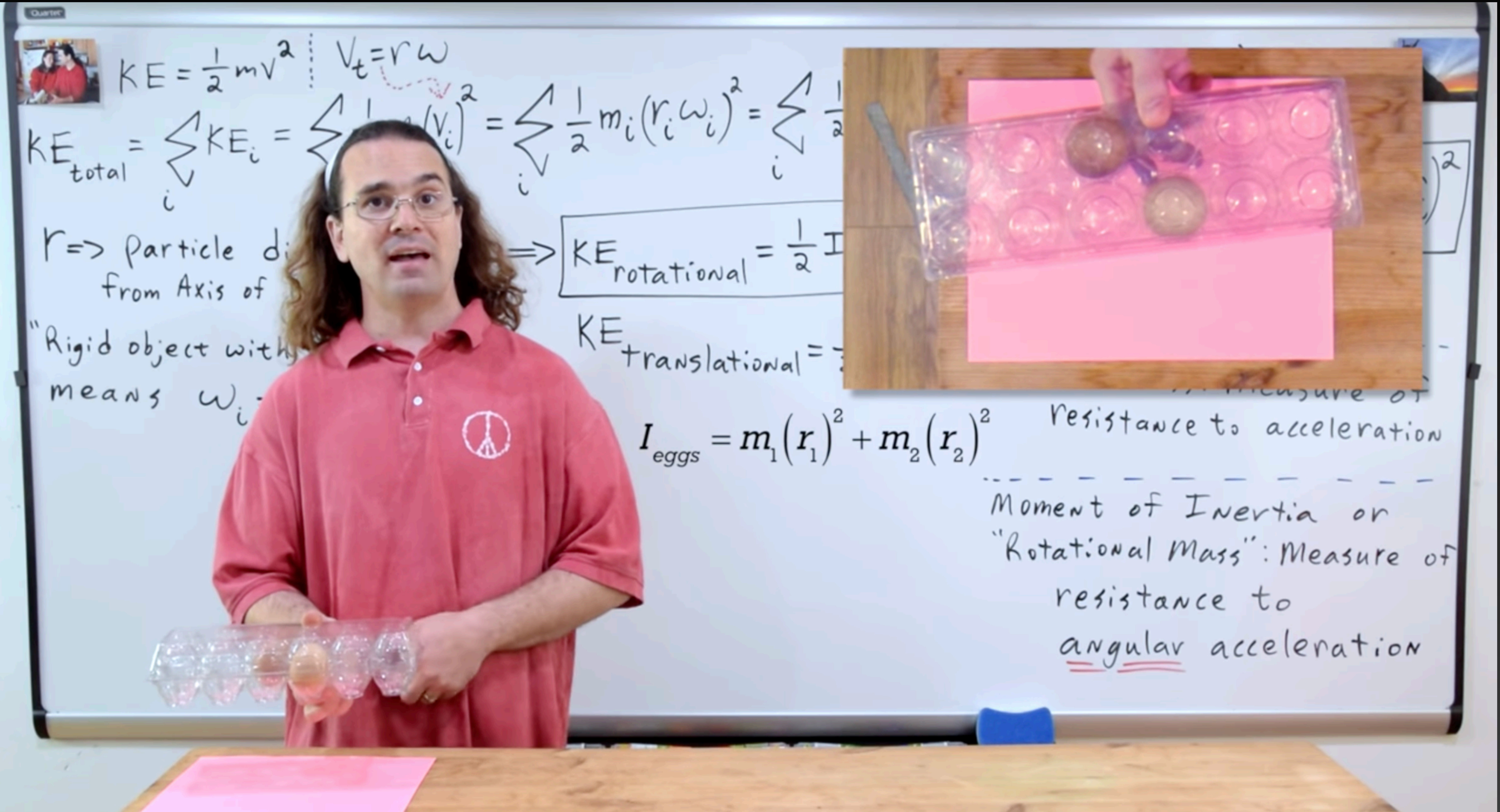
Rotational	Translational
$I = \sum_j m_j r_j^2$	m
$K = \frac{1}{2} I\omega^2$	$K = \frac{1}{2} mv^2$

Table 10.4 Rotational and Translational Kinetic Energies and Inertia

What is a moment of Inertia?

- We'll talk about this on Wednesday - for now, just think of it as "rotational mass", I

Deriving Rotational Kinetic Energy



Work in Rotational Motion

WORK-ENERGY THEOREM FOR ROTATION

The work-energy theorem for a rigid body rotating around a fixed axis is

$$W_{AB} = K_B - K_A \tag{10.29}$$

where

$$K = \frac{1}{2}I\omega^2$$

and the rotational work done by a net force rotating a body from point *A* to point *B* is

$$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i\right) d\theta. \tag{10.30}$$

Example

EXAMPLE 10.17

Rotational Work and Energy

A $12.0 \, N \cdot m$ torque is applied to a flywheel that rotates about a fixed axis and has a moment of inertia of $30.0 \, kg \cdot m^2$. If the flywheel is initially at rest, what is its angular velocity after it has turned through eight revolutions?

Strategy

We apply the work-energy theorem. We know from the problem description what the torque is and the angular displacement of the flywheel. Then we can solve for the final angular velocity.

Example

EXAMPLE 10.17

Rotational Work and Energy

A $12.0\,\text{N}\cdot\text{m}$ torque is applied to a flywheel that rotates about a fixed axis and has a moment of inertia of $30.0\,\text{kg}\cdot\text{m}^2$. If the flywheel is initially at rest, what is its angular velocity after it has turned through eight revolutions?

Strategy

We apply the work-energy theorem. We know from the problem description what the torque is and the angular displacement of the flywheel. Then we can solve for the final angular velocity.

Solution

The flywheel turns through eight revolutions, which is 16π radians. The work done by the torque, which is constant and therefore can come outside the integral in Equation 10.30, is

$$W_{AB} = \tau(\theta_B - \theta_A).$$

We apply the work-energy theorem:

$$W_{AB} = \tau(\theta_B - \theta_A) = \frac{1}{2}I\omega_B^2 - \frac{1}{2}I\omega_A^2.$$

With $\tau = 12.0\,\text{N}\cdot\text{m},\ \theta_B - \theta_A = 16.0\pi\,\text{rad},\ I = 30.0\,\text{kg}\cdot\text{m}^2,$ and $\omega_A = 0$, we have

$$12.0\,\text{N-m}(16.0\pi\,\text{rad}) = \frac{1}{2}(30.0\,\text{kg}\cdot\text{m}^2)(\omega_B^2) - 0.$$

Therefore,

$$\omega_B = 6.3\,\text{rad/s}.$$

This is the angular velocity of the flywheel after eight revolutions.

Significance

The work-energy theorem provides an efficient way to analyze rotational motion, connecting torque with rotational kinetic energy.

Power in Rotational Motion

$$P = \tau\omega. \quad (10.31)$$

to rotational motion. From Work and Kinetic Energy, the instantaneous power (or just power) is defined as the rate of doing work,

$$P = \frac{dW}{dt}.$$

If we have a constant net torque, Equation 10.25 becomes $W = \tau\theta$ and the power is

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\frac{d\theta}{dt}$$

Rotational Analogues

Rotational	Translational
	$\sum_i \vec{\mathbf{F}}_i = m\vec{\mathbf{a}}$
	$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$
	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

Rotational Analogues

Rotational	Translational
$\sum_i \tau_i = I\alpha$	$\sum_i \vec{\mathbf{F}}_i = m\vec{\mathbf{a}}$
$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta$	$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$
$P = \tau\omega$	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

Key Equations

Angular position	$\theta = \frac{s}{r}$
Angular velocity	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
Tangential speed	$v_t = r\omega$
Angular acceleration	$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Tangential acceleration	$a_t = r\alpha$
Average angular velocity	$\bar{\omega} = \frac{\omega_0 + \omega_f}{2}$
Angular displacement	$\theta_f = \theta_0 + \bar{\omega}t$
Angular velocity from constant angular acceleration	$\omega_f = \omega_0 + \alpha t$
Angular velocity from displacement and constant angular acceleration	$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
Change in angular velocity	$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$
Total acceleration	$\vec{\mathbf{a}} = \vec{\mathbf{a}}_c + \vec{\mathbf{a}}_t$

Key Equations

Rotational kinetic energy	$K = \frac{1}{2}\left(\sum_j m_j r_j^2\right)\omega^2$
Moment of inertia	$I = \sum_j m_j r_j^2$
Rotational kinetic energy in terms of the moment of inertia of a rigid body	$K = \frac{1}{2}I\omega^2$
Moment of inertia of a continuous object	$I = \int r^2 dm$
Parallel-axis theorem	$I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2$
Moment of inertia of a compound object	$I_{\text{total}} = \sum_i I_i$

Key Equations

Torque vector	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Magnitude of torque	$\lvert\vec{\boldsymbol{\tau}}\rvert = r_{\perp} F$
Total torque	$\vec{\tau}_{\text{net}} = \sum_i \lvert\vec{\tau}_i\rvert$
Newton's second law for rotation	$\sum_i \tau_i = I\alpha$
Incremental work done by a torque	$dW = \left(\sum_i \tau_i\right) d\theta$
Work-energy theorem	$W_{AB} = K_B - K_A$
Rotational work done by net force	$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i\right) d\theta$
Rotational power	$P = \tau\omega$

See you next class!

Attribution

This resource was significantly adapted from the Open Stax Instructor Slides provided by Rice University. It is released under a CC-BY 4.0 license.