Physics 111 - Class 9A
PE & Energy Conservation

October 31, 2022

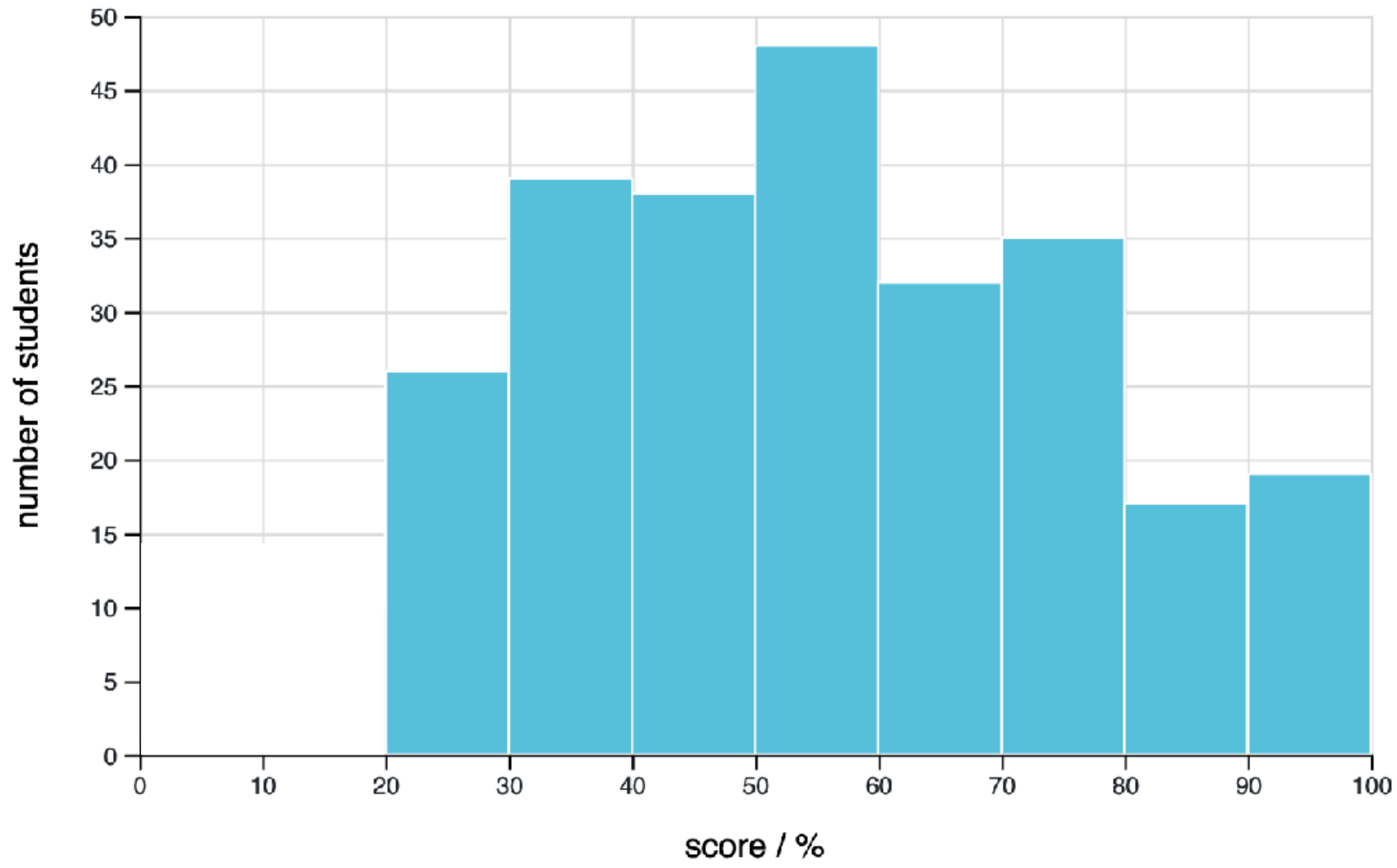
Class Outline

- Logistics / Announcements
- Test 3 Reflection
- Chapter 8 Section Summary
- ~~Clicker Questions (No time for CQs today!)~~
- Worked Problems

Logistics/Announcements

- Lab this week: Lab 6
- HW8 due this week on Thursday at 6 PM
- Learning Log 8 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Bonus Test 3 is this Friday!

Test 3 Reflection



Number of students	265
Mean score	54%

- Test 3 was pretty "tough but fair"
    - Several questions from previous homework assignments
    - Time wasn't a factor (on average) ; median: 36 mins
- More conceptual questions than usual, because understanding Forces is very important
- A few misconceptions...

Object in Circular Motion

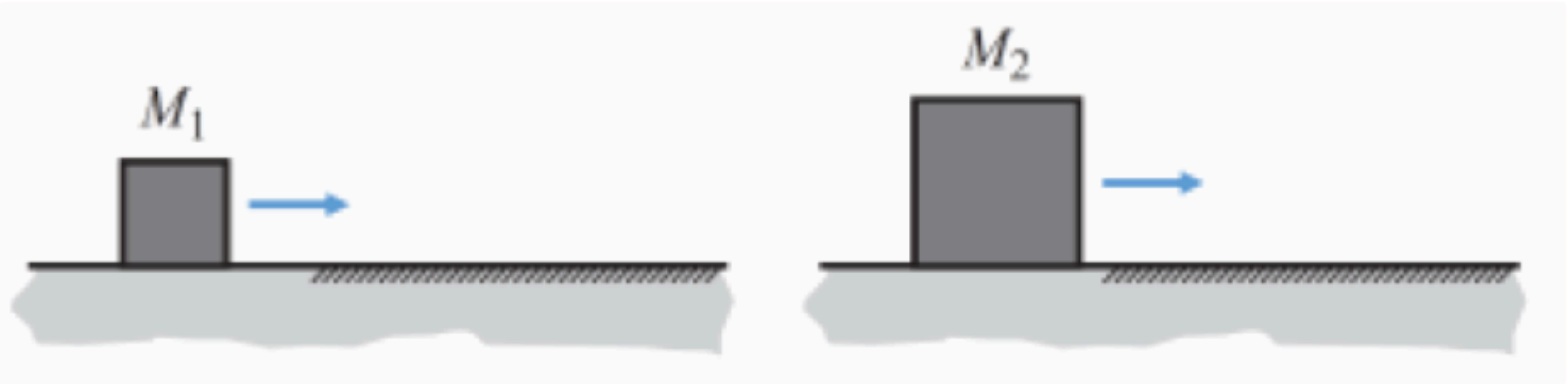
Object In Circular Motion

Which of the following statements is always true about an object in circular motion?

- (a) The object's net acceleration is towards the centre of the circular motion
- (b) The net force on the object has a non-zero component towards the centre of the circle
- (c) The object's angular velocity is given by v^2/r where r is the radius of the circle
- (d) The object has constant kinetic energy

Which Block stops first?

Two blocks, $M_1 < M_2$, having the same initial speed, move from a frictionless surface onto a surface having a coefficient of kinetic friction μ_k.



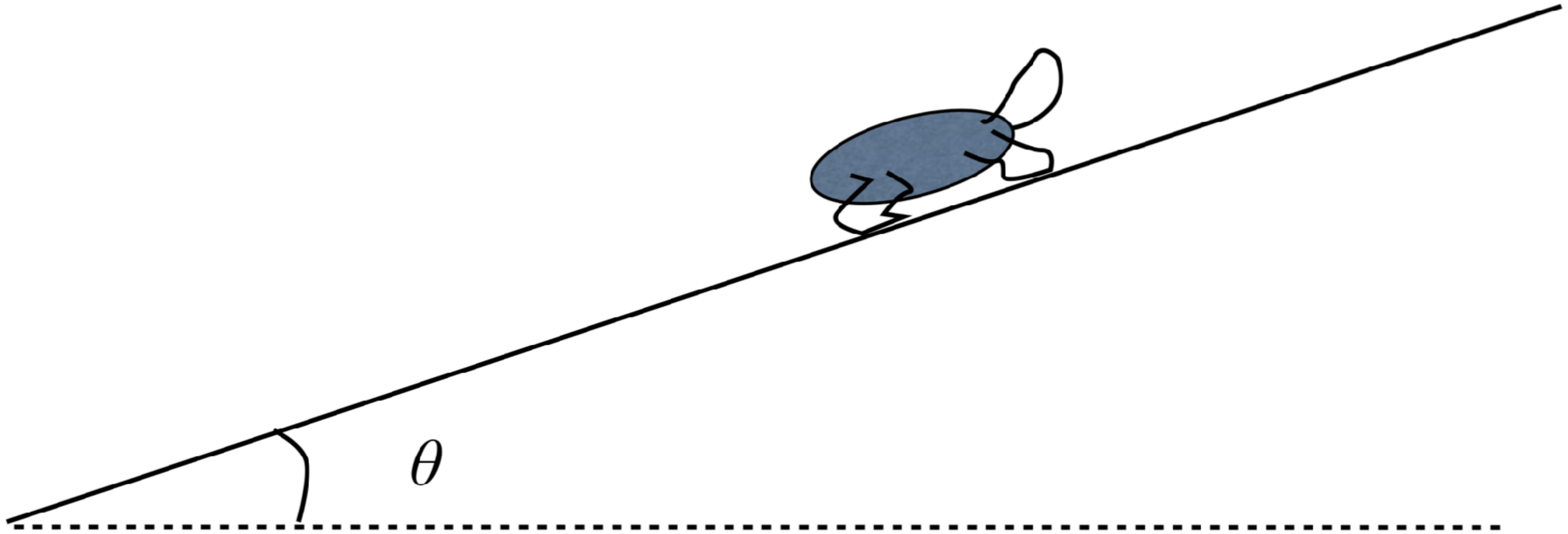
Which stops in a shorter distance?

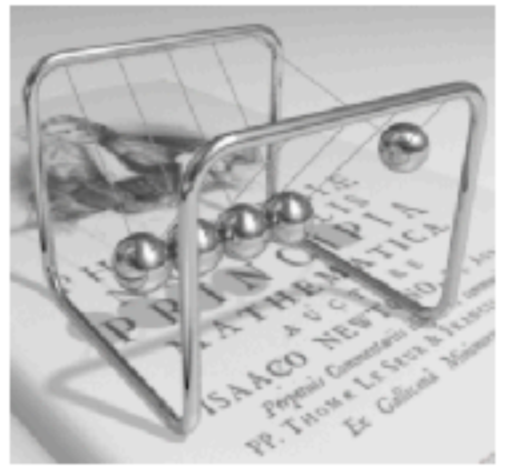
- (a) M_1
- (b) M_2
- (c) Cannot be determined without more information
- (d) Both stop in the same distance

Turtle on a Log

A turtle lies on a log in the sun as shown in the figure below. The turtle has mass m, the log makes an angle θ with respect to the horizontal and the coefficient of static friction between the turtle and the log is μ_s (where $\mu_s > \tan\theta$).

The magnitudes of the normal force, n and the frictional force f_s are:





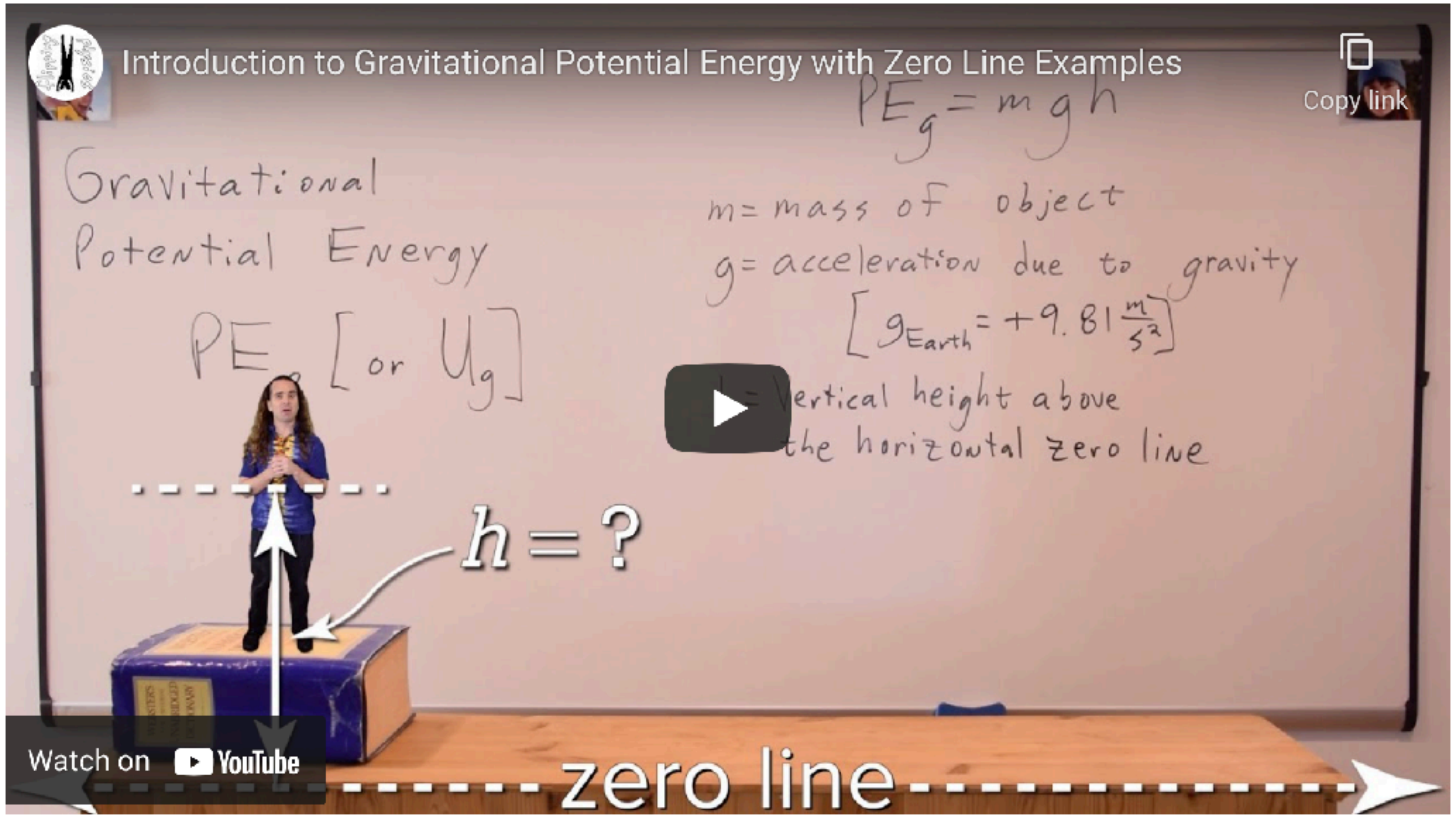
Physics 111

Videos

Below are the assigned videos for this week. The videos are collapsible so once you're done with one, you can move to the next one. In the sidebar on the right, you can use the checklists to keep track of what's done.

Required Videos

1. Introduction to Gravitational Potential Energy with Zero Line Examples



- Notes
- Direct link to Mr. P's page

Checklist of items

- [x] Video 1
- [x] Video 2
- [ ] Video 3
- [ ] Video 4
- [x] Video 5
- [ ] Video 6
- [ ] Video 7
- [x] Video 8
- [ ] Video 9
- [ ] Video 10

Table of contents

Search this book

My highlights

Figure 8.1 Shown here is part of a Ball Machine sculpture by George Rhoads. A ball in this contraption is lifted, rolls, falls, bounces, and collides with various objects, but throughout its travels, its kinetic energy changes in definite, predictable amounts, which depend on its position and the objects with which it interacts. (credit: modification of work by Roland Tanglao)

Chapter Outline

In George Rhoads' rolling ball sculpture, the principle of conservation of energy governs the changes in the ball's kinetic energy and relates them to changes and transfers for other types of energy associated with the ball's interactions. In this chapter, we introduce the important concept of potential energy. This will enable us to formulate

Monday's Class

8.1 Potential Energy of a System
8.4 Potential Energy Diagrams and Stability
8.5 Sources of Energy

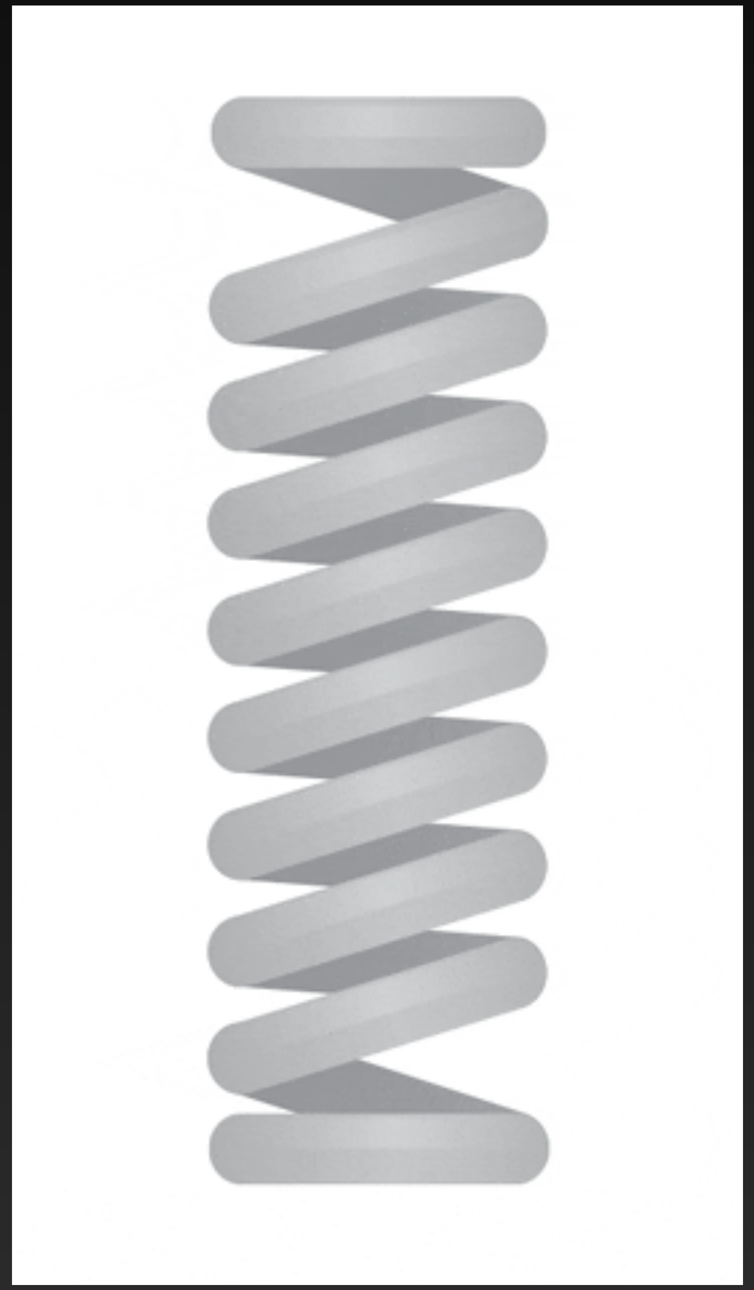
Potential Energy

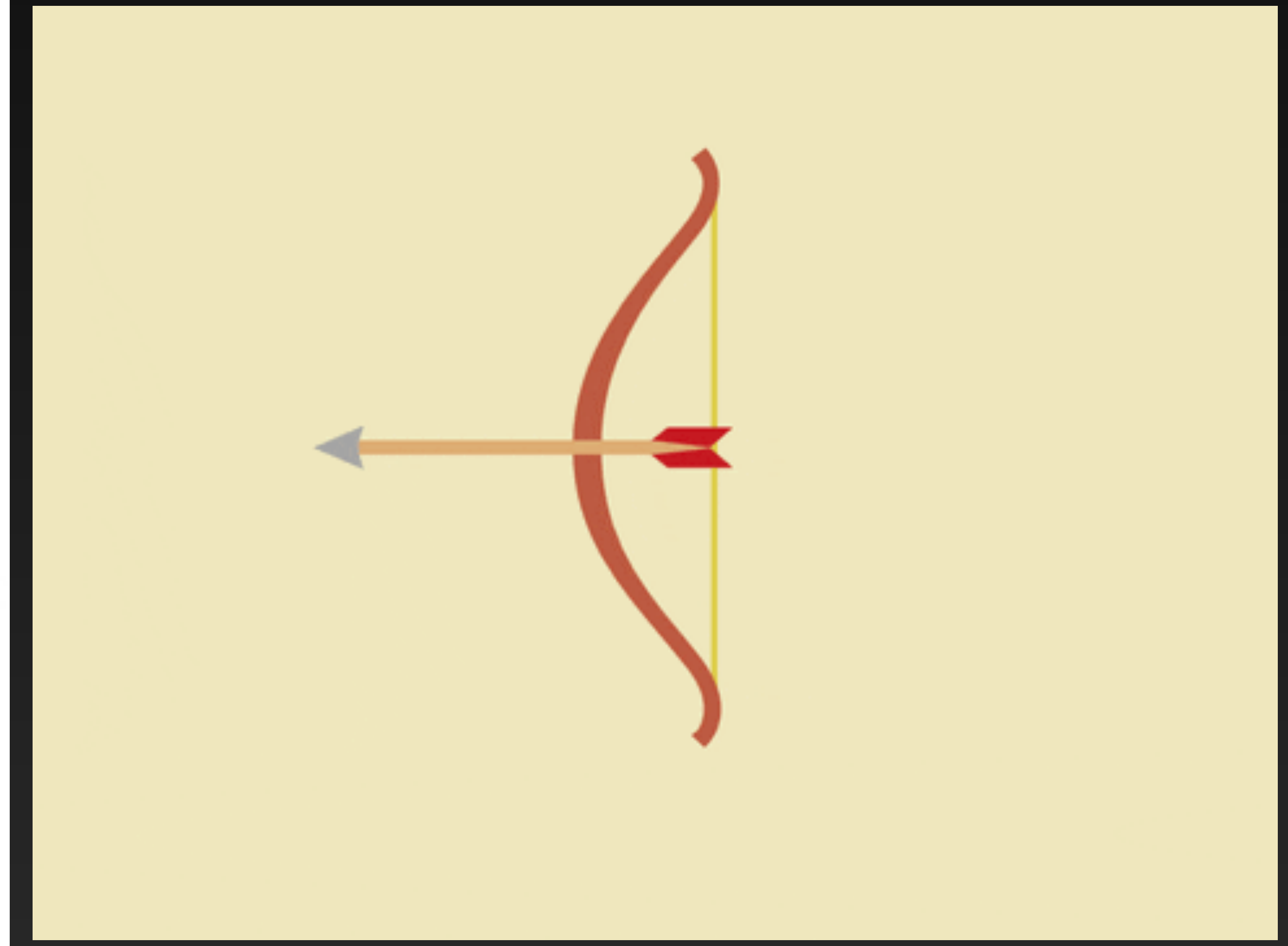
- Recall that "Kinetic Energy" was a characteristic of the object's **mass** and **velocity**.
- "Potential Energy" is a different form of energy that's characteristic of the object's **position**.
- There are different forms of "Potential Energy": Gravitational, Elastic/spring, Electrical, Nuclear…

Examples of Potential Energy

Gravitational Potential Energy

Elastic Potential Energy





Nuclear Potential Energy

Definition of Potential Energy

Based on this scenario, we can define the difference of potential energy from point A to point B as the negative of the work done:

$$\Delta U_{AB} = U_B - U_A = -W_{AB}. \qquad \boxed{8.1}$$

Definition of Potential Energy from Work

Work is done whenever an applied (external) force causes displacement.

$$W = \int \overrightarrow{F} \cdot d\hat{r}$$

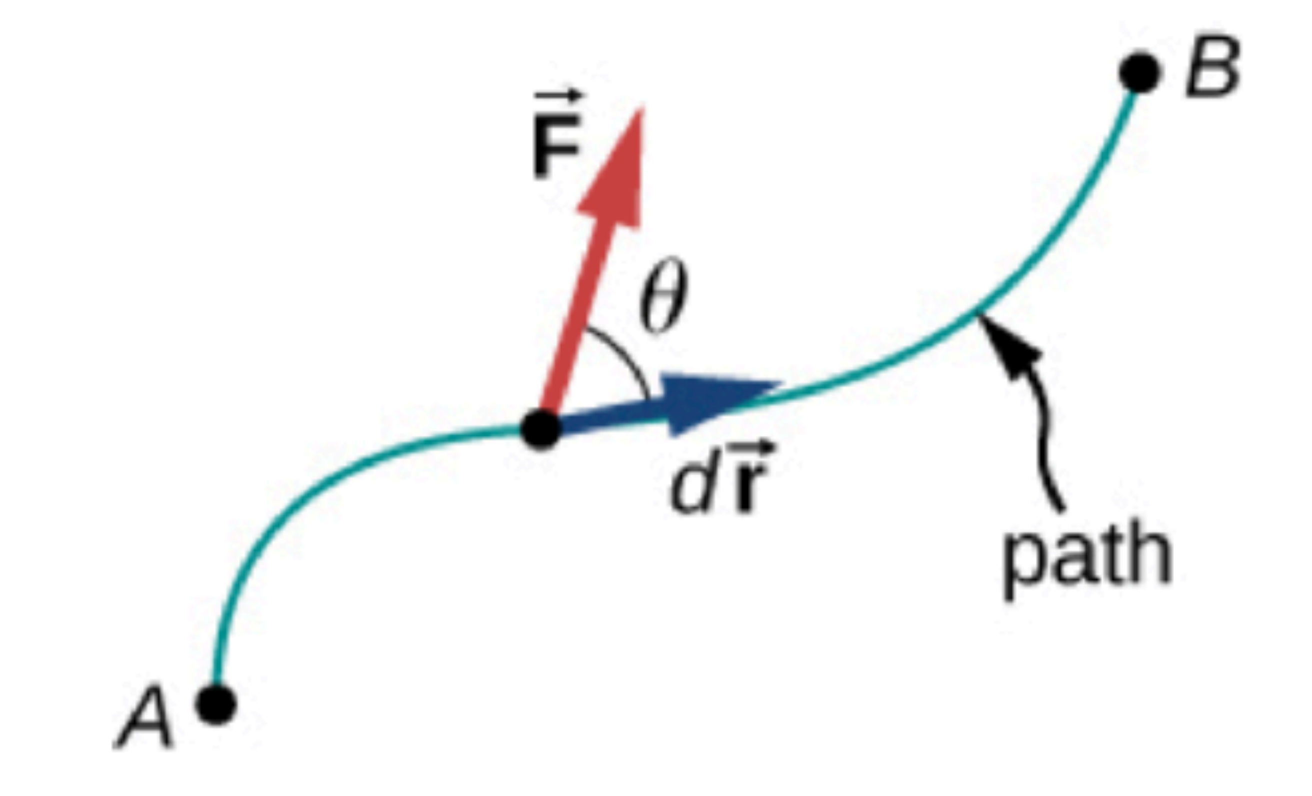


Figure 7.2 Vectors used to define work. The force acting on a particle and its infinitesimal displacement are shown at one point along the path between A and B. The infinitesimal work is the dot product of these two vectors; the total work is the integral of the dot product along the path.

Change in Potential Energy is the negative of the work done...

$$\Delta U_{AB} = - W_{AB}$$

EXAMPLE 8.2

Gravitational Potential Energy of a Hiker

The summit of Great Blue Hill in Milton, MA, is 147 m above its base and has an elevation above sea level of 195 m (Figure 8.3). (Its Native American name, *Massachusett*, was adopted by settlers for naming the Bay Colony and state near its location.) A 75-kg hiker ascends from the base to the summit. What is the gravitational potential energy of the hiker-Earth system with respect to zero gravitational potential energy at base height, when the hiker is (a) at the base of the hill, (b) at the summit, and (c) at sea level, afterward?

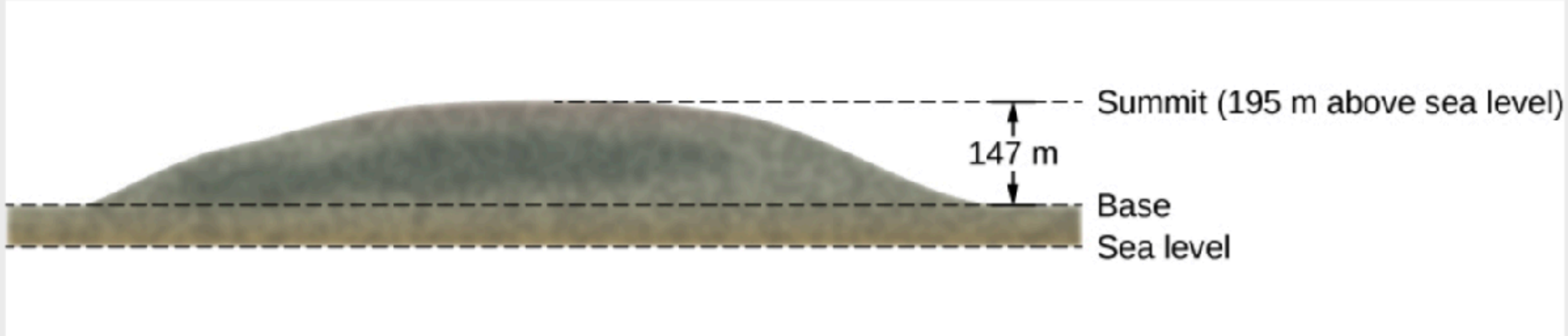


Figure 8.3 Sketch of the profile of Great Blue Hill, Milton, MA. The altitudes of the three levels are indicated.

Example 8.2

EXAMPLE 8.2

Gravitational Potential Energy of a Hiker

The summit of Great Blue Hill in Milton, MA, is 147 m above its base and has an elevation above sea level of 195 m (Figure 8.3). (Its Native American name, *Massachusett*, was adopted by settlers for naming the Bay Colony and state near its location.) A 75-kg hiker ascends from the base to the summit. What is the gravitational potential energy of the hiker-Earth system with respect to zero gravitational potential energy at base height, when the hiker is (a) at the base of the hill, (b) at the summit, and (c) at sea level, afterward?

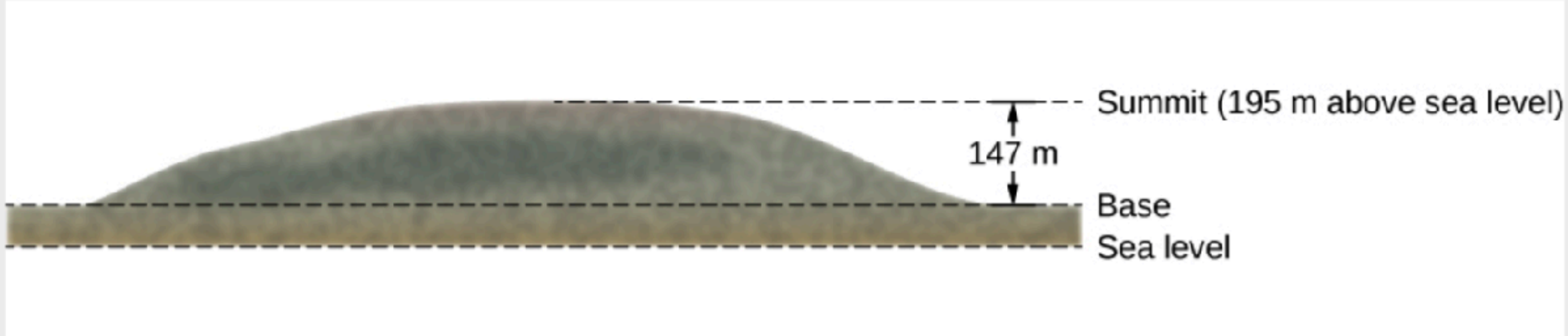


Figure 8.3 Sketch of the profile of Great Blue Hill, Milton, MA. The altitudes of the three levels are indicated.

Strategy

First, we need to pick an origin for the *y*-axis and then determine the value of the constant that makes the potential energy zero at the height of the base. Then, we can determine the potential energies from Equation 8.5, based on the relationship between the zero potential energy height and the height at which the hiker is located.


Introduction to Gravitational Potential Energy with Zero Line Examples
Copy link
Gravitational
Potential Energy
PE_g [or U_g]
PE_g = mgh
m = mass of object
g = acceleration due to gravity
[g_Earth = +9.81 m/s^2]
h = Vertical height above
the horizontal zero line

EXAMPLE 8.1

Basic Properties of Potential Energy

A particle moves along the *x*-axis under the action of a force given by $F = -ax^2$, where $a = 3\ \mathrm{N/m^2}$. (a) What is the difference in its potential energy as it moves from $x_A = 1\ \mathrm{m}$ to $x_B = 2\ \mathrm{m}$? (b) What is the particle's potential energy at $x = 1\ \mathrm{m}$ with respect to a given 0.5 J of potential energy at $x = 0$?

EXAMPLE 8.1

Basic Properties of Potential Energy

A particle moves along the x-axis under the action of a force given by $F = -ax^2$, where $a = 3\ \text{N/m}^2$. (a) What is the difference in its potential energy as it moves from $x_A = 1\ \text{m}$ to $x_B = 2\ \text{m}$? (b) What is the particle's potential energy at $x = 1\ \text{m}$ with respect to a given 0.5 J of potential energy at $x = 0$?

Solution

a. The work done by the given force as the particle moves from coordinate x to $x + dx$ in one dimension is

$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = F dx = -ax^2 dx.$$

Substituting this expression into Equation 8.1, we obtain

$$\Delta U = -W = \int_{x_1}^{x_2} ax^2 dx = \frac{1}{3}(3\ \text{N/m}^2)x^3\Big|_{1\ \text{m}}^{2\ \text{m}} = 7\ \text{J}.$$

b. The indefinite integral for the potential energy function in part (a) is

$$U(x) = \frac{1}{3}ax^3 + \text{const.},$$

and we want the constant to be determined by

$$U(0) = 0.5\ \text{J}.$$

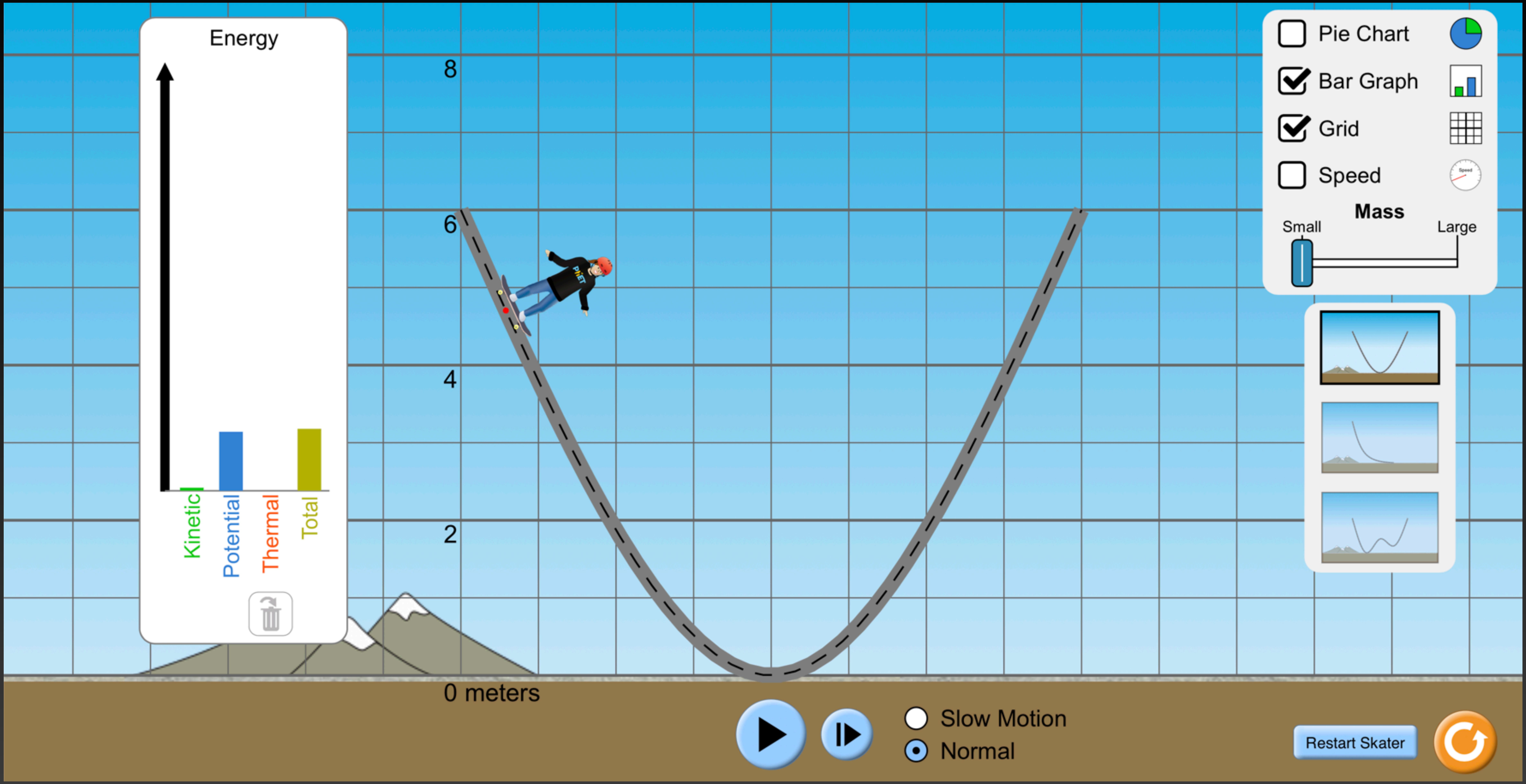
Thus, the potential energy with respect to zero at $x = 0$ is just

$$U(x) = \frac{1}{3}ax^3 + 0.5\ \text{J}.$$

Therefore, the potential energy at $x = 1\ \text{m}$ is

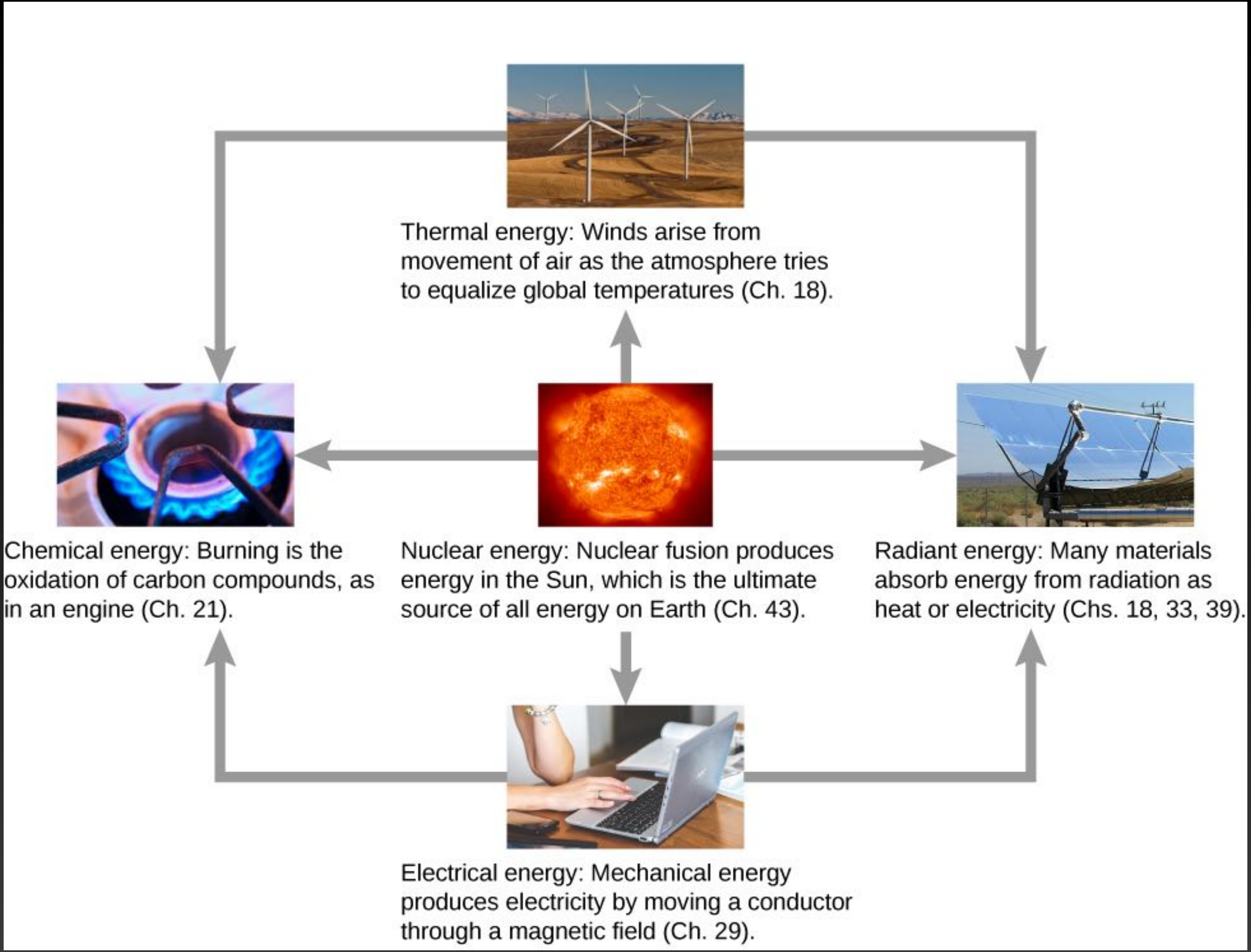
$$U(1\ \text{m}) = \frac{1}{3}\left(3\ \text{N/m}^2\right)(1\ \text{m})^3 + 0.5\ \text{J} = 1.5\ \text{J}.$$

Energy Skatepark



Energy Skatepark

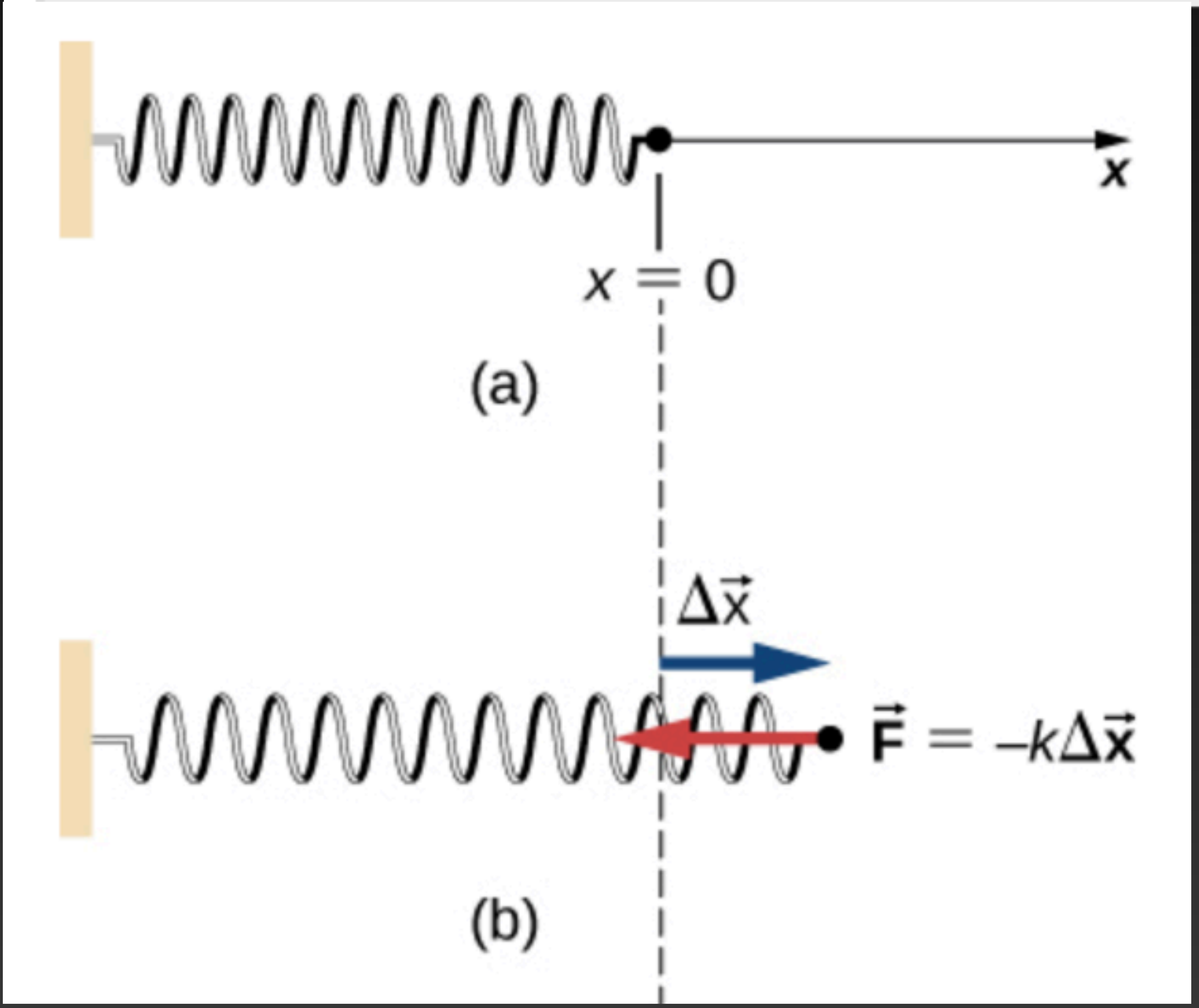
Energy Types



EXAMPLE 7.5

Work Done by a Spring Force

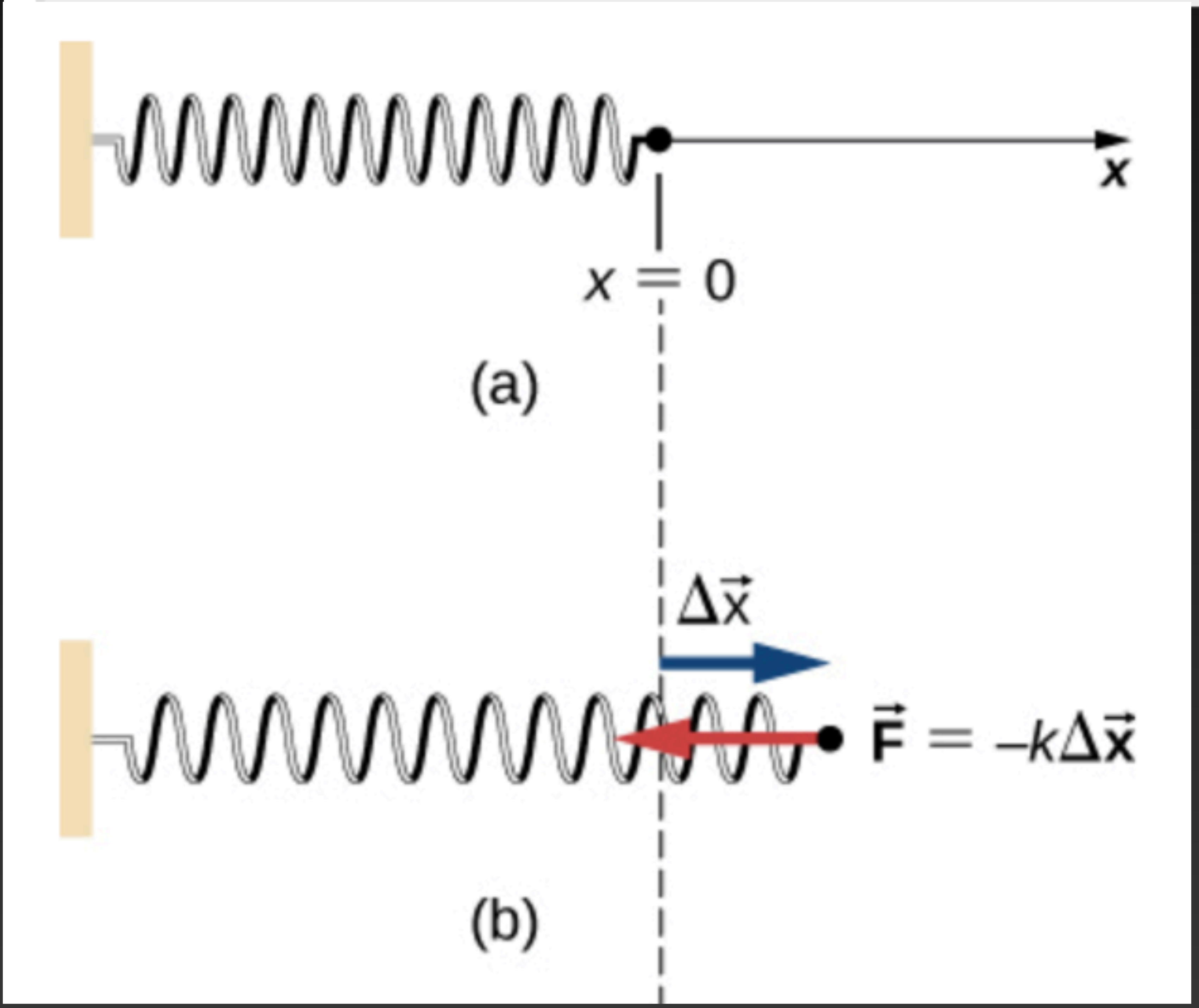
A perfectly elastic spring requires 0.54 J of work to stretch 6 cm from its equilibrium position, as in Figure 7.7(b). (a) What is its spring constant *k*? (b) How much work is required to stretch it an additional 6 cm?



EXAMPLE 7.5

Work Done by a Spring Force

A perfectly elastic spring requires 0.54 J of work to stretch 6 cm from its equilibrium position, as in Figure 7.7(b). (a) What is its spring constant *k*? (b) How much work is required to stretch it an additional 6 cm?



Strategy

Work "required" means work done against the spring force, which is the negative of the work in Equation 7.5, that is

$$W = \frac{1}{2}k(x_B^2 - x_A^2).$$

For part (a), $x_A = 0$ and $x_B = 6\text{cm}$; for part (b), $x_B = 6\text{cm}$ and $x_B = 12\text{cm}$. In part (a), the work is given and you can solve for the spring constant; in part (b), you can use the value of *k*, from part (a), to solve for the work.

Solution

a. $W = 0.54\,\text{J} = \frac{1}{2}k[(6\text{ cm})^2 - 0]$, so $k = 3\text{ N/cm}$.
b. $W = \frac{1}{2}(3\text{ N/cm})[(12\text{ cm})^2 - (6\text{ cm})^2] = 1.62\,\text{J}$.

Significance

Since the work done by a spring force is independent of the path, you only needed to calculate the difference in the quantity $½kx^2$ at the end points. Notice that the work required to stretch the spring from 0 to 12 cm is four times that required to stretch it from 0 to 6 cm, because that work depends on the square of the amount of stretch from equilibrium, $½kx^2$. In this circumstance, the work to stretch the spring from 0 to 12 cm is also equal to the work for a composite path from 0 to 6 cm followed by an additional stretch from 6 cm to 12 cm. Therefore,

Elastic Potential Energy

Elastic potential energy

In Work, we saw that the work done by a perfectly elastic spring, in one dimension, depends only on the spring constant and the squares of the displacements from the unstretched position, as given in Equation 7.5. This work involves only the properties of a Hooke's law interaction and not the properties of real springs and whatever objects are attached to them. Therefore, we can define the difference of elastic potential energy for a spring force as the negative of the work done by the spring force in this equation, before we consider systems that embody this type of force. Thus,

$$\Delta U = -W_{AB} = \frac{1}{2}k(x_B^2 - x_A^2), \tag{8.6}$$

Example 8.3

EXAMPLE 8.3

Spring Potential Energy

A system contains a perfectly elastic spring, with an unstretched length of 20 cm and a spring constant of 4 N/cm. (a) How much elastic potential energy does the spring contribute when its length is 23 cm? (b) How much more potential energy does it contribute if its length increases to 26 cm?

Example 8.3

EXAMPLE 8.3

Spring Potential Energy

A system contains a perfectly elastic spring, with an unstretched length of 20 cm and a spring constant of 4 N/cm. (a) How much elastic potential energy does the spring contribute when its length is 23 cm? (b) How much more potential energy does it contribute if its length increases to 26 cm?

Strategy

When the spring is at its unstretched length, it contributes nothing to the potential energy of the system, so we can use Equation 8.7 with the constant equal to zero. The value of *x* is the length minus the unstretched length. When the spring is expanded, the spring's displacement or difference between its relaxed length and stretched length should be used for the *x*-value in calculating the potential energy of the spring.

Solution

a. The displacement of the spring is $x = 23\text{ cm} - 20\text{ cm} = 3\text{ cm}$, so the contributed potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{2}(4\text{ N/cm})\,(3\text{ cm})^2 = 0.18\text{ J}$.
b. When the spring's displacement is $x = 26\text{ cm} - 20\text{ cm} = 6\text{ cm}$, the potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{2}(4\text{ N/cm})\,(6\text{ cm})^2 = 0.72\text{ J}$, which is a 0.54-J increase over the amount in part (a).

Example 8.3

EXAMPLE 8.3

Spring Potential Energy

A system contains a perfectly elastic spring, with an unstretched length of 20 cm and a spring constant of 4 N/cm. (a) How much elastic potential energy does the spring contribute when its length is 23 cm? (b) How much more potential energy does it contribute if its length increases to 26 cm?

Strategy

When the spring is at its unstretched length, it contributes nothing to the potential energy of the system, so we can use Equation 8.7 with the constant equal to zero. The value of *x* is the length minus the unstretched length. When the spring is expanded, the spring's displacement or difference between its relaxed length and stretched length should be used for the *x*-value in calculating the potential energy of the spring.

Solution

a. The displacement of the spring is $x = 23\text{ cm} - 20\text{ cm} = 3\text{ cm}$, so the contributed potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{2}(4\text{ N/cm})\,(3\text{ cm})^2 = 0.18\text{ J}$.
b. When the spring's displacement is $x = 26\text{ cm} - 20\text{ cm} = 6\text{ cm}$, the potential energy is $U = \frac{1}{2}kx^2 = \frac{1}{2}(4\text{ N/cm})\,(6\text{ cm})^2 = 0.72\text{ J}$, which is a 0.54-J increase over the amount in part (a).

Significance

Calculating the elastic potential energy and potential energy differences from Equation 8.7 involves solving for the potential energies based on the given lengths of the spring. Since *U* depends on x^2, the potential energy for a compression (negative *x*) is the same as for an extension of equal magnitude.

EXAMPLE 8.1

Basic Properties of Potential Energy

A particle moves along the *x*-axis under the action of a force given by $F = -ax^2$, where $a = 3\ \mathrm{N/m^2}$. (a) What is the difference in its potential energy as it moves from $x_A = 1\ \mathrm{m}$ to $x_B = 2\ \mathrm{m}$? (b) What is the particle's potential energy at $x = 1\ \mathrm{m}$ with respect to a given 0.5 J of potential energy at $x = 0$?

EXAMPLE 8.1

Basic Properties of Potential Energy

A particle moves along the *x*-axis under the action of a force given by $F = -ax^2$, where $a = 3\ \text{N/m}^2$. (a) What is the difference in its potential energy as it moves from $x_A = 1\ \text{m}$ to $x_B = 2\ \text{m}$? (b) What is the particle's potential energy at $x = 1\ \text{m}$ with respect to a given 0.5 J of potential energy at $x = 0$?

Solution

a. The work done by the given force as the particle moves from coordinate x to $x + dx$ in one dimension is

$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = F dx = -ax^2 dx.$$

Substituting this expression into Equation 8.1, we obtain

$$\Delta U = -W = \int_{x_1}^{x_2} ax^2 dx = \frac{1}{3}(3\ \text{N/m}^2)x^3\Big|_{1\ \text{m}}^{2\ \text{m}} = 7\ \text{J}.$$

b. The indefinite integral for the potential energy function in part (a) is

$$U(x) = \frac{1}{3}ax^3 + \text{const.},$$

and we want the constant to be determined by

$$U(0) = 0.5\ \text{J}.$$

Thus, the potential energy with respect to zero at $x = 0$ is just

$$U(x) = \frac{1}{3}ax^3 + 0.5\ \text{J}.$$

Therefore, the potential energy at $x = 1\ \text{m}$ is

$$U(1\ \text{m}) = \frac{1}{3}\left(3\ \text{N/m}^2\right)(1\ \text{m})^3 + 0.5\ \text{J} = 1.5\ \text{J}.$$

Key Equations

Difference of potential energy	$\Delta U_{AB} = U_B - U_A = -W_{AB}$
Potential energy with respect to zero of potential energy at $\vec{\mathbf{r}}_0$	$\Delta U = U\left(\vec{\mathbf{r}}\right) - U\left(\vec{\mathbf{r}}_0\right)$
Gravitational potential energy near Earth's surface	$U\left(y\right) = mgy + \text{const.}$
Potential energy for an ideal spring	$U\left(x\right) = \frac{1}{2}kx^2 + \text{const.}$
Work done by conservative force over a closed path	$W_{\text{closed path}} = \int \vec{\mathbf{F}}_{\text{cons}} \cdot d\vec{\mathbf{r}} = 0$
Condition for conservative force in two dimensions	$\left(\frac{dF_x}{dy}\right) = \left(\frac{dF_y}{dx}\right)$
Conservative force is the negative derivative of potential energy	$F_l = -\frac{dU}{dl}$
Conservation of energy with no non-conservative forces	$0 = W_{nc,AB} = \Delta(K+U)_{AB} = \Delta E_{AB}.$

Clicker Questions

CQ.9.1

Which activity requires a person to exert force on an object that causes the object to move but does not change the kinetic or potential energy of the object?

a) moving an object to a greater height with acceleration

b) moving an object to a greater height without acceleration

c) carrying an object with acceleration at the same height

d) carrying an object without acceleration at the same height

A

B

C

D

E

CQ.9.2

You are riding a bicycle up a gentle hill. It is fairly easy to increase your potential energy, but to increase your kinetic energy would be harder.

a) True

b) False

A

B

C

D

E

CQ.9.3

How much work is done by gravity when a 7.64 kg boulder falls to the ground from the top of a 33.4 m tall cliff?

a) 0.0 J, because gravity doesn't do work

b) 2.24 J

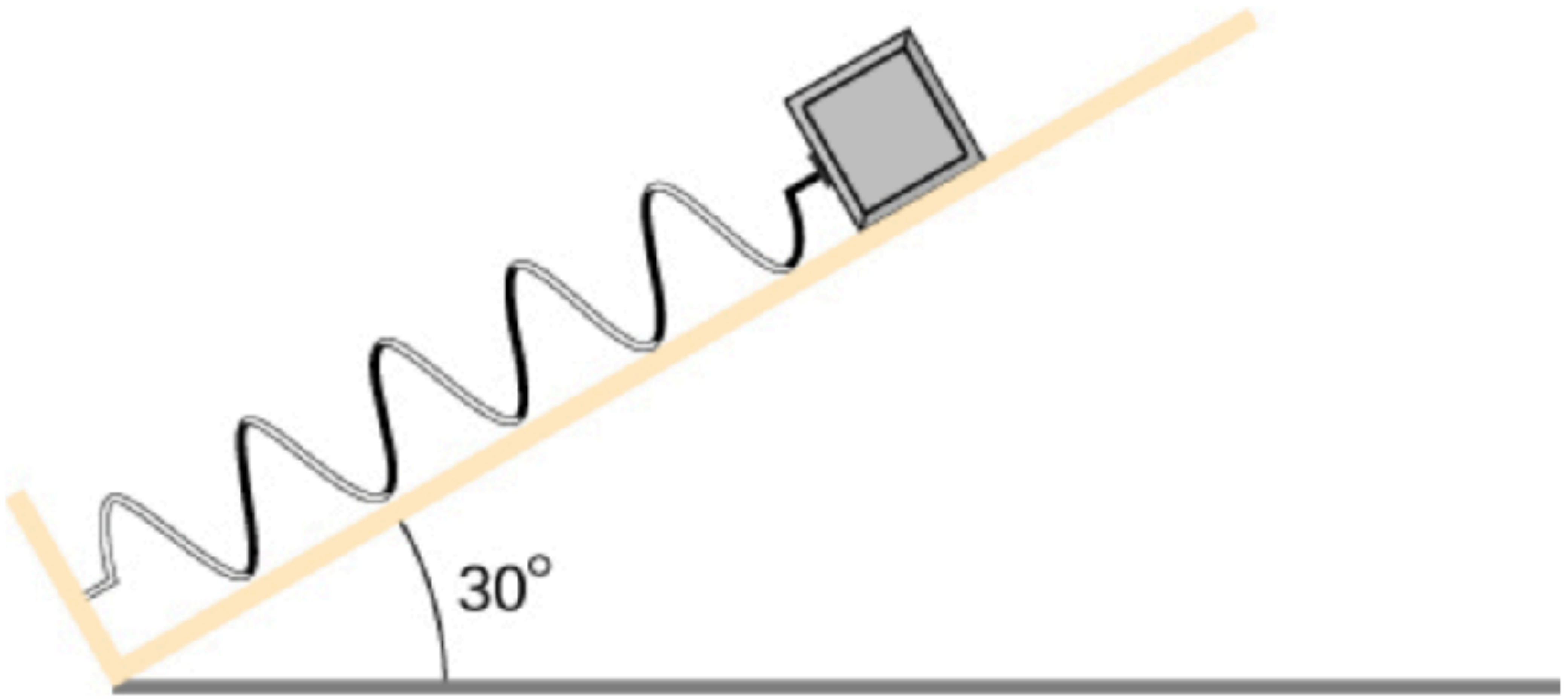
c) 26.0 J

d) 2500 J

A B C D E

Activity:
Worked Problems

64 . A block of mass 500 g is attached to a spring of spring constant 80 N/m (see the following figure). The other end of the spring is attached to a support while the mass rests on a rough surface with a coefficient of friction of 0.20 that is inclined at angle of $30°$. The block is pushed along the surface till the spring compresses by 10 cm and is then released from rest. (a) How much potential energy was stored in the block-spring-support system when the block was just released? (b) Determine the speed of the block when it crosses the point when the spring is neither compressed nor stretched. (c) Determine the position of the block where it just comes to rest on its way up the incline.



WP 9.1

See you next class!

Attribution