Week 8 Tutorial

Tutorial Section

Tutorial Time

Tutorial TA Name

Question 1

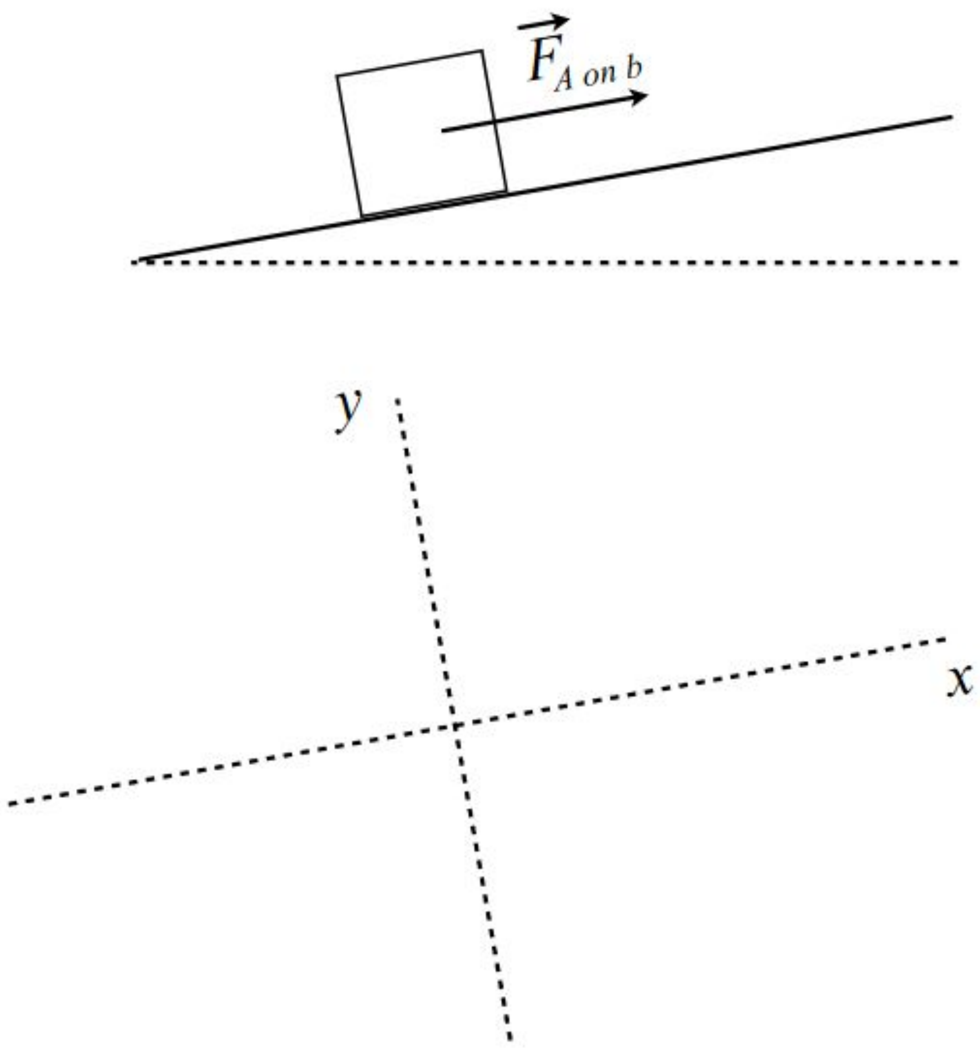

FIG. 1:

1. On moving day Adrienne pushes a box at constant speed up a ramp. She pushes parallel to the ramp with a force of magnitude $F_{\mathrm{A\ on\ b}}$. The ramp is 10° above the horizontal direction as shown in Fig. 1, the coefficients of friction between the box and the ramp are $\mu_s = 0.5$ and $\mu_k = 0.3$, and the box weighs 180 N.

Question 1

(a) Draw a free body diagram for this problem on the labelled axes provided on Fig. 1. Label each force with subscripts indicating the agent of the force and the force on which it acts. Use r for ramp, A for Adrienne, E for Earth and b for box. /8

(b) Is this an example of static or dynamic equilibrium? If so, explain which. /2

(c) From your free body diagram write Newton's second law equations in component form. /8

(d) Solve your equations to find $F_{\text{A on b}}$. /4

(e) If Adrienne first placed the box on the ramp at rest, how hard would Adrienne have had to initially push the box to just overcome the static force of friction? (Express your answer as $F_{\text{A on b}} >$ ________.) /2

(f) Once the box started moving, if Adrienne maintained the force you found in (e), what would have happened to the box? /2

Problem Solving Framework

PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH **16,** 010123 (2020)

Template for teaching and assessment of problem solving in introductory physics

E. W. Burkholder,[1,*] J. K. Miles,[2] T. J. Layden,[2] K. D. Wang,[3]
A. V. Fritz,[4] and C. E. Wieman[1,3]

1. Framing

- Visual Representation
- Relevant Concepts
- Similar Problems
- Assumptions and Simplifications
- Information Needed

2. Planning

- Solution Plan
- Rough Estimate

3. Execution

Carry-out Plan for solving
- Work in algebra/symbols until the BITTER end
- Plug in numbers at the LAST step

4. Answer Checking

- Compare to Estimate
- Limits Test
- Units Check
- Getting (UnStuck)

Reference: Template for teaching and assessment of problem solving in introductory physics

1. Framing

2. Planning

3. Execution

4. Answer Checking

1. Framing

- Visual representation: Draw a FBD that illustrates all the forces and their components
- Assumptions and simplifications: Only consider this motion in 2D.
- Relevant concept: Dynamic equilibrium, Newton's second and third law, Friction
- Information needed: Forces and their components, weight, coefficients of friction, angles
- Similar problems: Problems from week 7 tutorials, any problem involving dynamic equilibrium and friction

2. Planning

- Rough estimate: Draw the Free Body Diagram including every force and its components, include direction of acceleration and label your axes. Roughly estimate the expected magnitudes of the forces that you will calculate and their expected signs.
- Solution plan: Write the Newton's second law equations and solve for the values needed

3. Execution

Start by drawing the Free body diagram

(a) Draw a free body diagram for this problem on the labelled axes provided on Fig. 1. Label each force with subscripts indicating the agent of the force and the force on which it acts. Use r for ramp, A for Adrienne, E for Earth and b for box. /8

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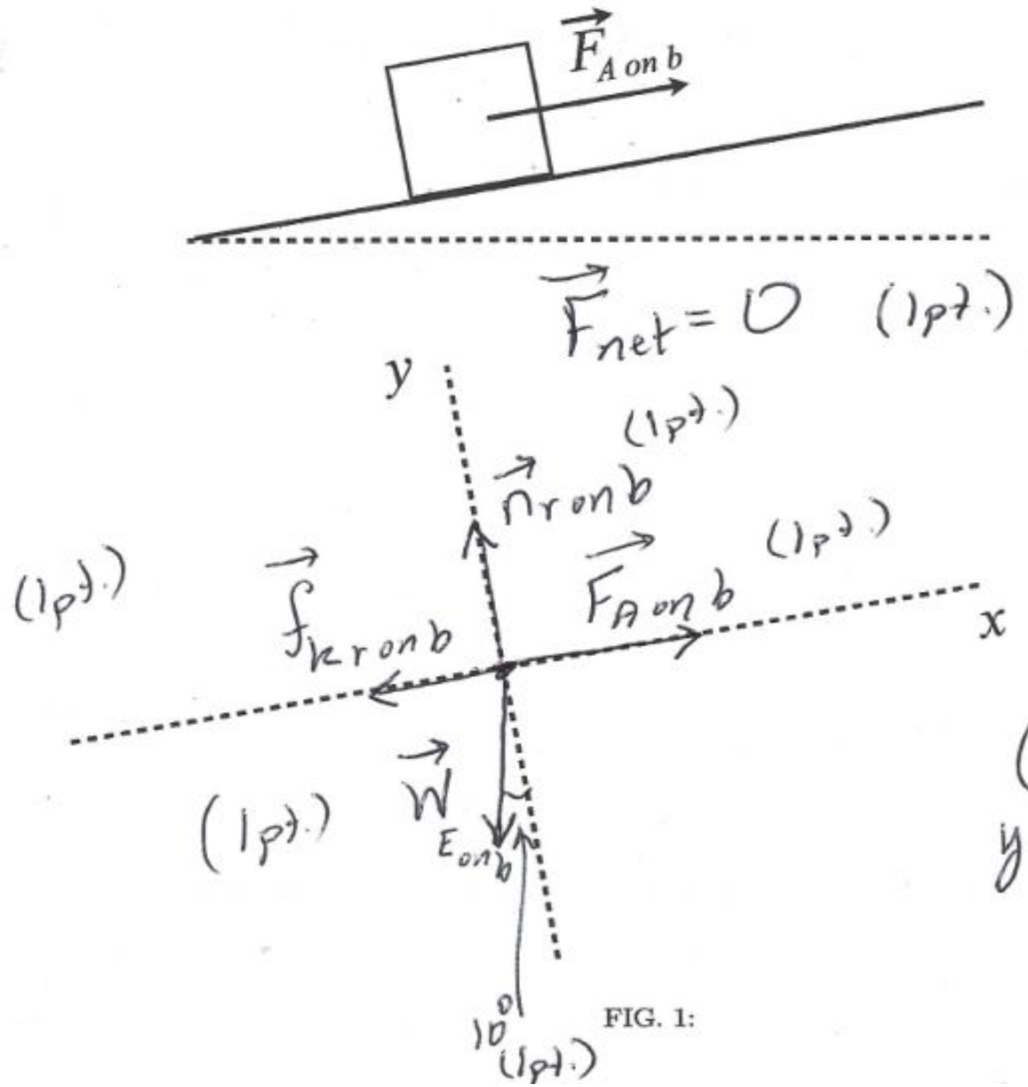

(1pt.) sum of x-components of forces should add to zero

(1pt.) sum of y-components of forces should add to zero.

FIG. 1:

(b) Is this an example of static or dynamic equilibrium? If so, explain which. /2

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This is an example of dynamic equilibrium.
(1pt.)

The system is moving at constant speed so it is in equilibrium but it is not static. (1pt)

(c) From your free body diagram write Newton's second law equations in component form. /8

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(1pt.) (1pt.) (1pt.) (1pt.) (½pt.)

Sum of forces in x: $F_{A\,on\,b} - f_{kr\,on\,b} - W_{E\,on\,b}\sin(10^\circ) = 0$

(1pt.) (1pt.) (1pt.) (½ pt.)

Sum of forces in y: $n_{r\,on\,b} - W_{E\,on\,b}\cos(10^\circ) = 0$

$\vec{F}_{net} = 0$ (1pt.)

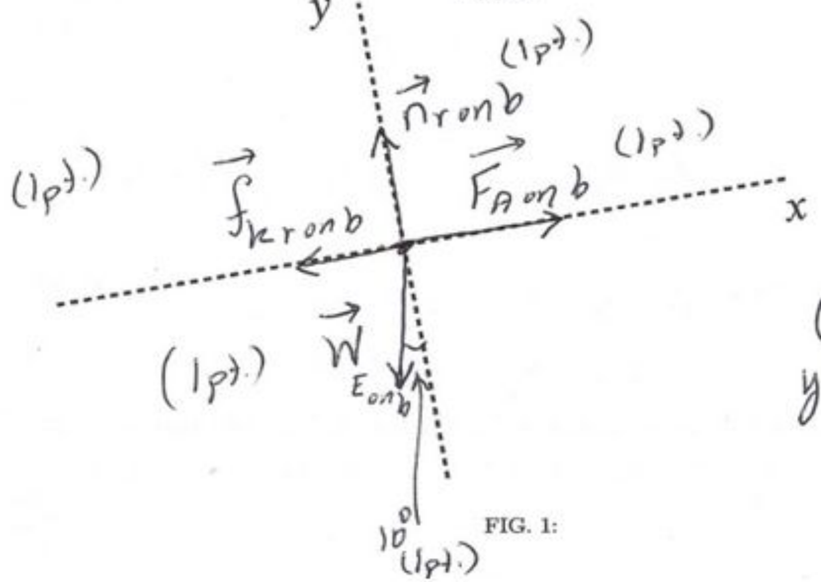

FIG. 1:

(−½ point per incorrect sign)

(d) Solve your equations to find $F_{\mathrm{A\ on\ b}}$. /4

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$$F_{A\,on\,b} = f_{k\,r\,on\,b} + W_{E\,on\,b}\sin 10^\circ \text{ (1pt.)}$$

$$= \mu_k W_{E\,on\,b}\cos 10^\circ + W_{E\,on\,b}\sin 10^\circ$$

(1pt.) either

$$= \left((0.3)\cos 10^\circ + \sin 10^\circ\right)(180\,N)$$

$$= 84\,N \quad \text{(1pt.)}$$

(e) If Adrienne first placed the box on the ramp at rest, how hard would Adrienne have had to initially push the box to just overcome the static force of friction? (Express your answer as $F_{\text{A on b}} >$ __________.) /2

(e) If Adrienne first placed the box on the ramp at rest, how hard would Adrienne have had to initially push the box to just overcome the static force of friction? (Express your answer as $F_{\text{A on b}} >$ 120 N.) /2

$$F_{A\ on\ b} > (\mu_s \cos 10^\circ + \sin 10^\circ)\, W_{E\ on\ b} \quad (1pt.)$$

$$= (0.5 \cos 10^\circ + \sin(10^\circ))\ 180\,N$$

$$= 120\,N \quad (1pt.)$$

(f) Once the box started moving, if Adrienne maintained the force you found in (e), what would have happened to the box? /2

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$F_{net_x} > 0$ (1pt.) so the box would have continued to speed up (1pt.) as Adrienne pushed it up the ramp.

4. Answer Checking

Compare to estimates: compare the calculation results to your rough estimates see whether the sign matches.

Units: Think about what unit we use for forces and angles in this problem and verify they are correct

Limits: Think about what would be some realistic values for the forces you calculated. Do your answers make sense? Do they seem WAY far off?

Getting (UnStuck)? If you get stuck, go to the next question and come back to this one later.

Question 2

The position of a vole as it runs toward a garden, is given by the expression $x(t) = t^3 - t + 10$ where $x(t)$ is position expressed as a function of time. The units of $x(t)$ are cm, and the units of t are seconds.

t [s]	$x(t)$ [cm]
-2	
-1	
$-1/\sqrt{3}$	10.385
0	
$1/\sqrt{3}$	9.615
1	
2	

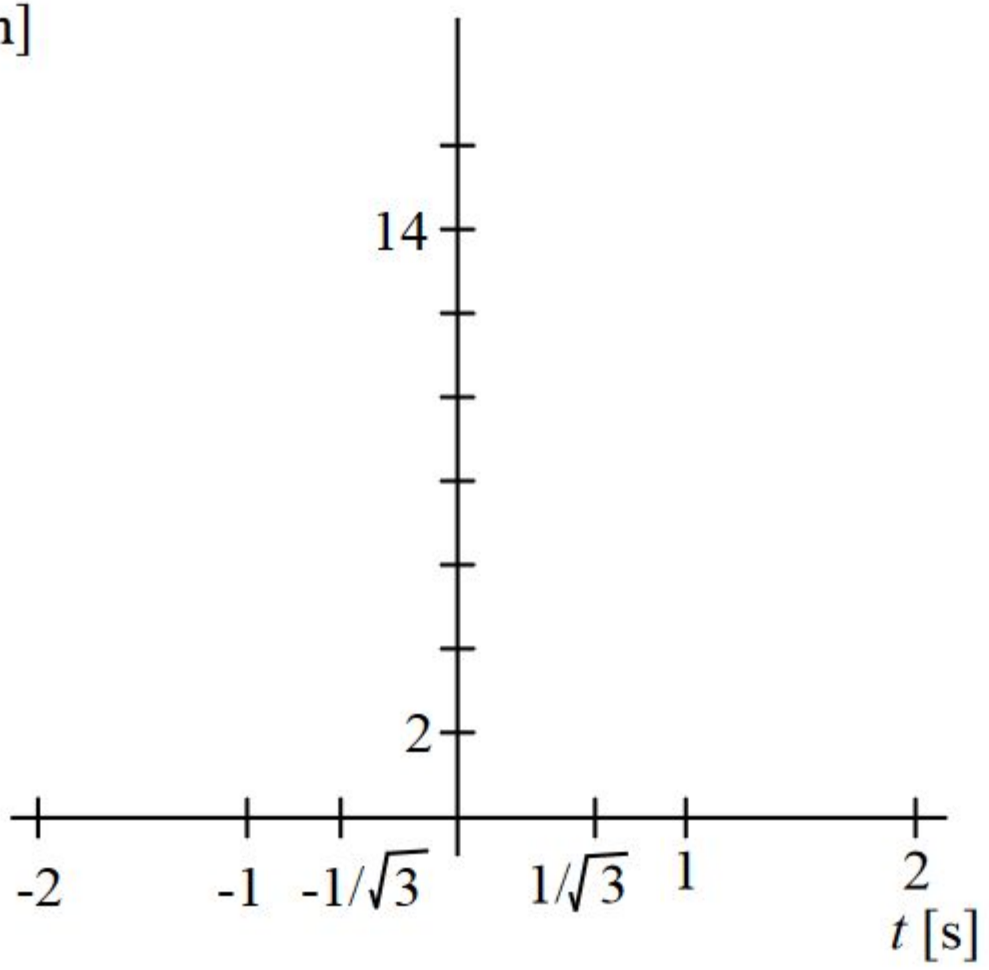

FIG. 1:

The position of a vole as it runs toward a garden, is given by the expression $x(t) = t^3 - t + 10$ where $x(t)$ is position expressed as a function of time. The units of $x(t)$ are cm, and the units of t are seconds.

1. (a) In the table shown on the left of Fig. 1 list values of $x(t)$ for each given time.

(b) On the coordinate system shown in Fig. 1 plot the location of each of the points from the table and connect them with a smooth line.

(c) At what times does the vole stop moving? (Show your calculations including finding an expression for the velocity, $v_x(t)$, and explain). /4

(d) What does it mean for t to be less than zero? /2

(e) Between the times $t = -2$ s and $t = 2$ s, state any intervals over which the velocity is negative, and over which the velocity is positive. /3

(f) Find an expression for the acceleration, $a_x(t)$.

(g) Between the times $t = -2$ s and $t = 2$ s, state any intervals over which the vole is speeding up, and any intervals of time over which it is slowing down. /3

1. Framing

2. Planning

3. Execution

4. Answer Checking

1. Framing

Visual representation: You can draw a qualitative diagram/sketch of the situation if you want, but the graphs are usually plenty

Assumptions and simplifications: Only consider this motion in 1D.

Relevant concept: the relationship between position x, velocity Vx: Vx=dx/dt, integrals and derivatives

Information needed: position change during certain time period

Similar problems: given Vx vs t graph, draw ax vs t graph; given Vx vs t graph, draw x vs t graph, etc.

2. Planning

- Rough estimate: We can estimate the sign of velocity (positive/negative/0) from visually viewing the graph, and relative change velocity based on the slope.

  We can also visualize the situation a bit and predict what it might look like.

- Solution plan: Plug in the values of t in the x(t) formula, fill in the table and sketch each point on the graph.

3. Execution

- Plug in values of t into x(t) and sketch each point

The position of a vole as it runs toward a garden, is given by the expression $x(t) = t^3 - t + 10$ where $x(t)$ is position expressed as a function of time. The units of $x(t)$ are cm, and the units of t are seconds.

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-1	
$-1/\sqrt{3}$	10.385
0	
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1	
2	

The position of a vole as it runs toward a garden, is given by the expression $x(t) = t^3 - t + 10$ where $x(t)$ is position expressed as a function of time. The units of $x(t)$ are cm, and the units of t are seconds.

1. (a) In the table shown on the left of Fig. 1 list values of $x(t)$ for each given time.

t [s]	$x(t)$ [cm]
-2	4 (1/2 pt. each)
-1	10
$-1/\sqrt{3}$	10.385
0	10
$1/\sqrt{3}$	9.615
1	10
2	16

Plug each value of t into the given formula: $x(t) = t^3 - t + 10$

i.e. For $t = -2$:

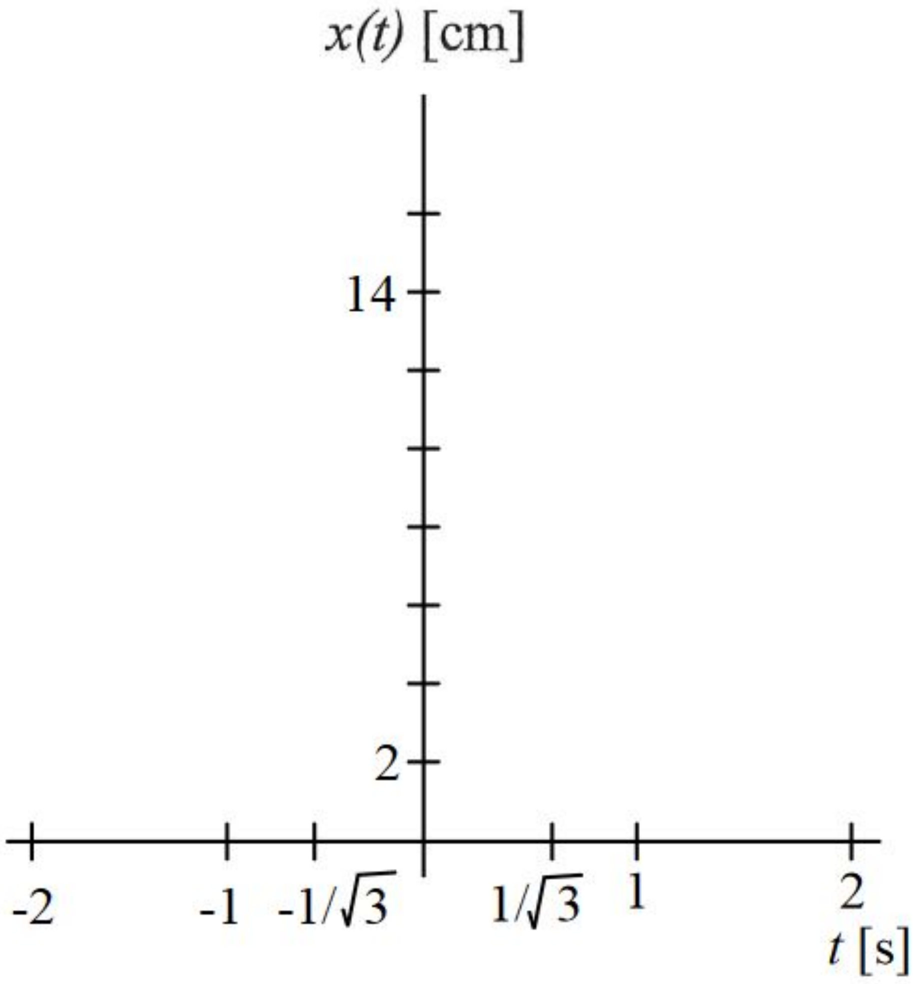
$$x(-2) = (-2)^3 - (-2) + 10$$

$$x(-2) = (-8) + 2 + 10$$

$$x(-2) = 4$$

(b) On the coordinate system shown in Fig. 1 plot the location of each of the points from the table and connect them with a smooth line.

/6



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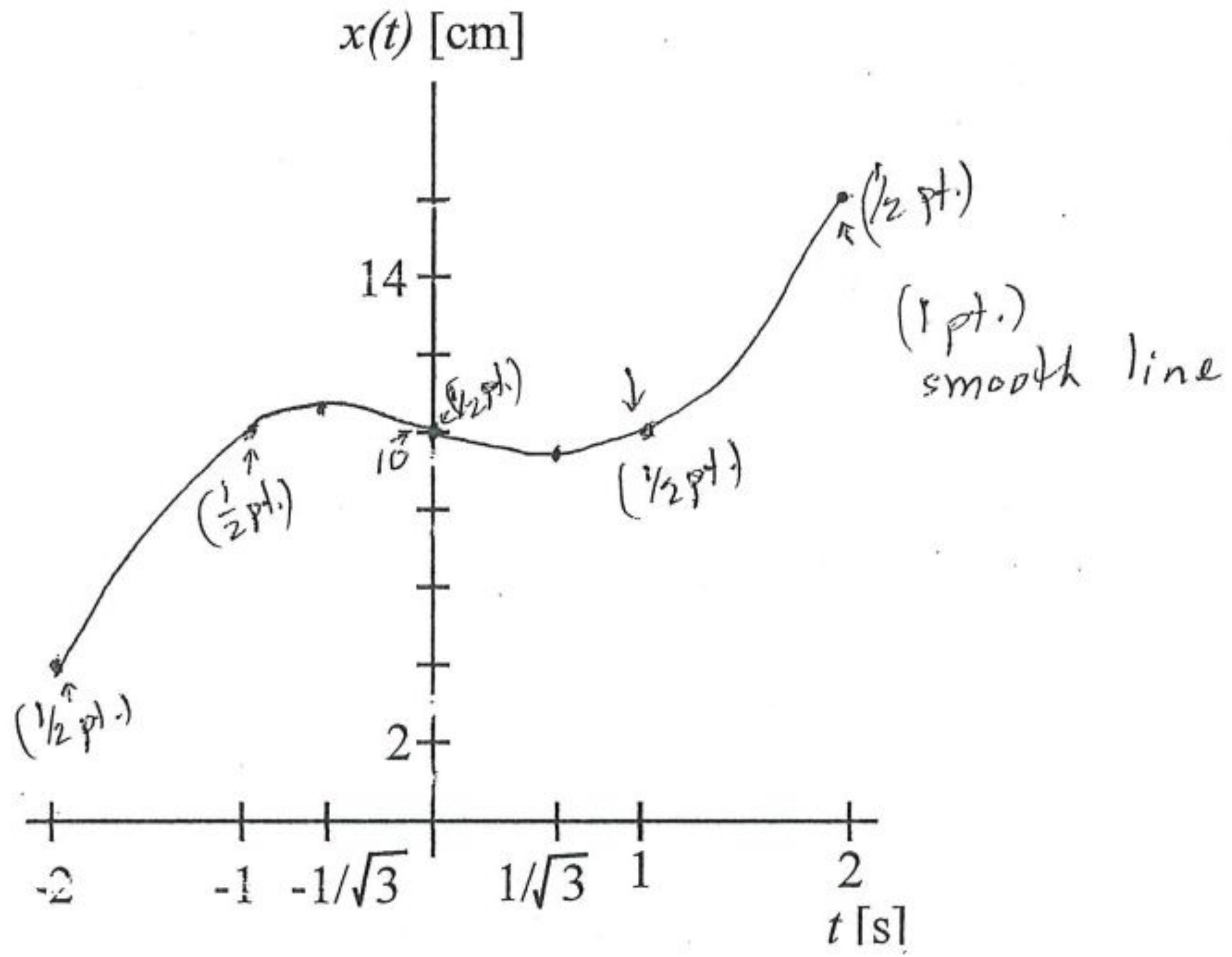
(b) On the coordinate system shown in Fig. 1 plot the location of each of the points from the table and connect them with a smooth line.

/6



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t [s]	$x(t)$ [cm]
-2	4 (1/2 pt. each)
-1	10
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1	10
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(c) At what times does the vole stop moving? (Show your calculations including finding an expression for the velocity, $v_x(t)$, and explain). /4

(c) At what times does the vole stop moving? (Show your calculations including finding an expression for the velocity, $v_x(t)$, and explain). /4

$$x(t) = t^3 - t + 10$$

Apply the Sum/Difference Rule: $(f \pm g)' = f' \pm g'$

$$= \frac{\partial}{\partial t}(t^3) - \frac{\partial}{\partial t}(t) + \frac{\partial}{\partial t}(10)$$

$\frac{\partial}{\partial t}(t^3) = 3t^2$ $\frac{\partial}{\partial t}(t) = 1$ $\frac{\partial}{\partial t}(10) = 0$

$$\frac{\partial}{\partial t}(t^3 - t + 10) = 3t^2 - 1$$

$V_x(t) = \underset{(1pt.)}{3t^2} - \underset{(1pt)}{1}$

$V_x(t) = 0$ when $\underset{(1pt.)}{3t^2 = 1}$ so the vole stops moving at both $\underset{(1pt.)}{t = +1/\sqrt{3}\,s}$ and $t = -1/\sqrt{3}\,s$.

(d) What does it mean for t to be less than zero? /2

(d) What does it mean for t to be less than zero? /2

$t<0$ means an earlier time (1pt.) than the time at which we set the clock. Here the vole takes a brief pause in their forward motion at $t=0$ where we have chosen to place the origin (1pt.) of our clock.

Essentially, it just means that a negative time is before the point of time we chose to label as zero. For example, whenever we launch a rocket we call the time of lift-off T = 0 s, and everything before that is often referred to as something in the format of T - 12 seconds.

We are doing the same here except with the vole's slight pause instead of lift-off.

(e) Between the times $t = -2$ s and $t = 2$ s, state any intervals over which the velocity is negative, and over which the velocity is positive. /3

(e) Between the times $t = -2$ s and $t = 2$ s, state any intervals over which the velocity is negative, and over which the velocity is positive. /3

$V_x < 0$ for $-\frac{1}{\sqrt{3}}\,s < t < \frac{1}{\sqrt{3}}\,s$ (1 pt.)

$V_x > 0$ for $t < -1/\sqrt{3}$ s (1 pt.)

$V_x >$ for $t > 1/\sqrt{3}$ s (1 pt)

t [s]	$x(t)$ [cm]
-2	4 (1/2 pt. each)
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0	10
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1	10
2	16

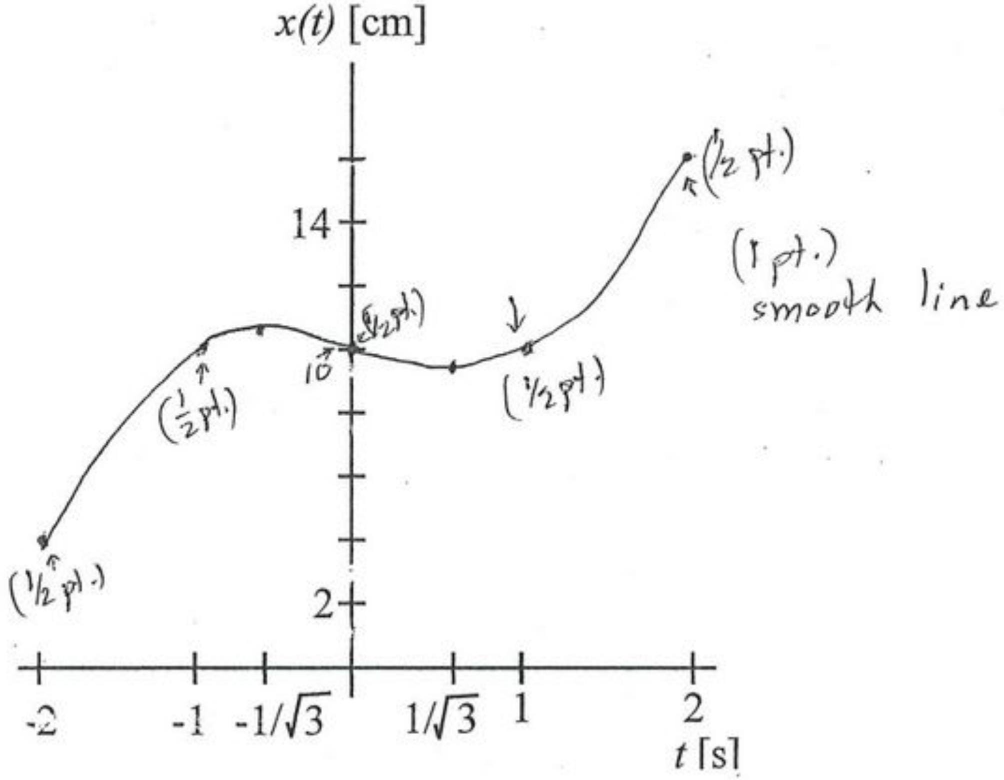

FIG. 1:

(f) Find an expression for the acceleration, $a_x(t)$. /2

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$$\frac{dv_x}{dt} = a_x$$

Apply the Sum/Difference Rule: $(f \pm g)' = f' \pm g'$

$$= \frac{\partial}{\partial t}\left(3t^2\right) - \frac{\partial}{\partial t}(1)$$

$\frac{\partial}{\partial t}\left(3t^2\right) = 6t$ $\frac{\partial}{\partial t}(1) = 0$

$$= \frac{\partial}{\partial t}\left(3t^2 - 1\right) = 6t$$

$a_x = 6t$ (1pt.) The acceleration changes sign at $t = 0s$. (1pt.)

(g) Between the times $t = -2$ s and $t = 2$ s, state any intervals over which the vole is speeding up, and any intervals of time over which it is slowing down. /3

(g) Between the times $t = -2$ s and $t = 2$ s, state any intervals over which the vole is speeding up, and any intervals of time over which it is slowing down. /3

The vole speeds up $-\frac{1}{\sqrt{3}}s < t < 0s$ and $\frac{1}{\sqrt{3}} < t$

(1/2 pt.) (1/2 pt.)

and slows down for

$t < -\frac{1}{\sqrt{3}}$ and $0 < t < \frac{1}{\sqrt{3}}$

(1/2 pt.) (1/2 pt.)

(1pt.) expressed as inequality.

4. Answer Checking

Compare to estimates: compare the calculation results to your rough estimates see whether the sign matches.

Units: Think about what unit we use for Vx or ax in this problem and add them in your final answer.

Limits: Think about how x vs t graph looks like with 0 velocity? How increasing or decreasing the velocity change x vs t graph?

Getting (UnStuck)? If you get stuck, go to the next question and come back to this one later.