You can draw here

Physics 111 - Class 12C
Fixed Axis Rotation

Do not draw in/on this box!

November 26, 2021

You can draw here

You can draw here

Class Outline

- Logistics / Announcements
- Homework Reflection
- Chapter 10 Section Summary
- Worked Problems

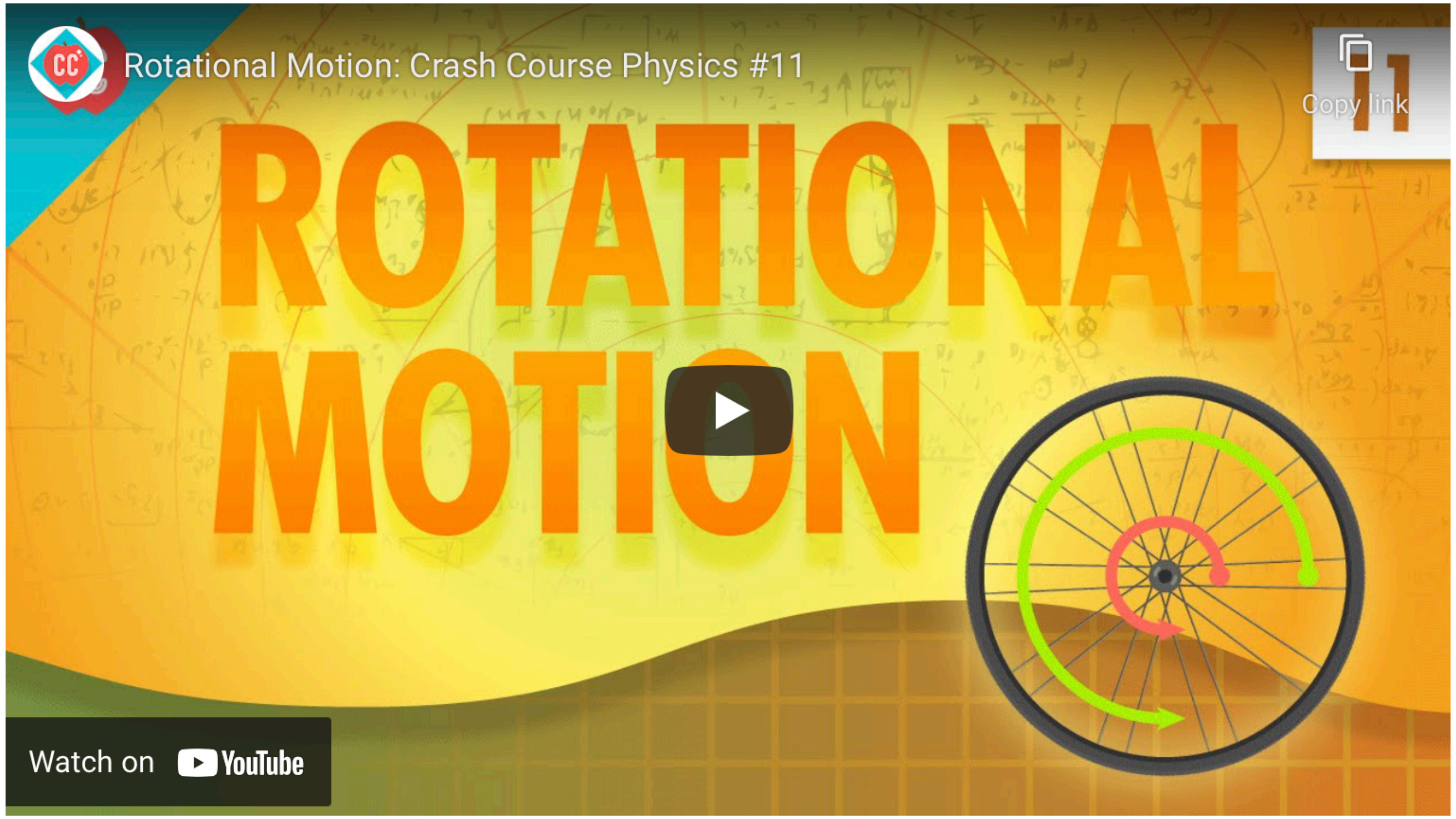
Logistics/Announcements

- Lab this week: Lab 8
- HW10 due this week on Thursday at 6 PM
- Learning Log 10 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Test 5 available this week (Chapters 8 & 9)
  - Test Window: Friday 6 PM - Sunday 6 PM
  - Last one!

Physics 111

Search this book...

Rotational Motion



Torque

Wed

Fri

Figure 10.1 Brazos wind farm in west Texas. During 2019, wind farms in the United States had an average power output of 34 gigawatts, which is enough to power 28 million homes. (credit: modification of work by U.S. Department of Energy)

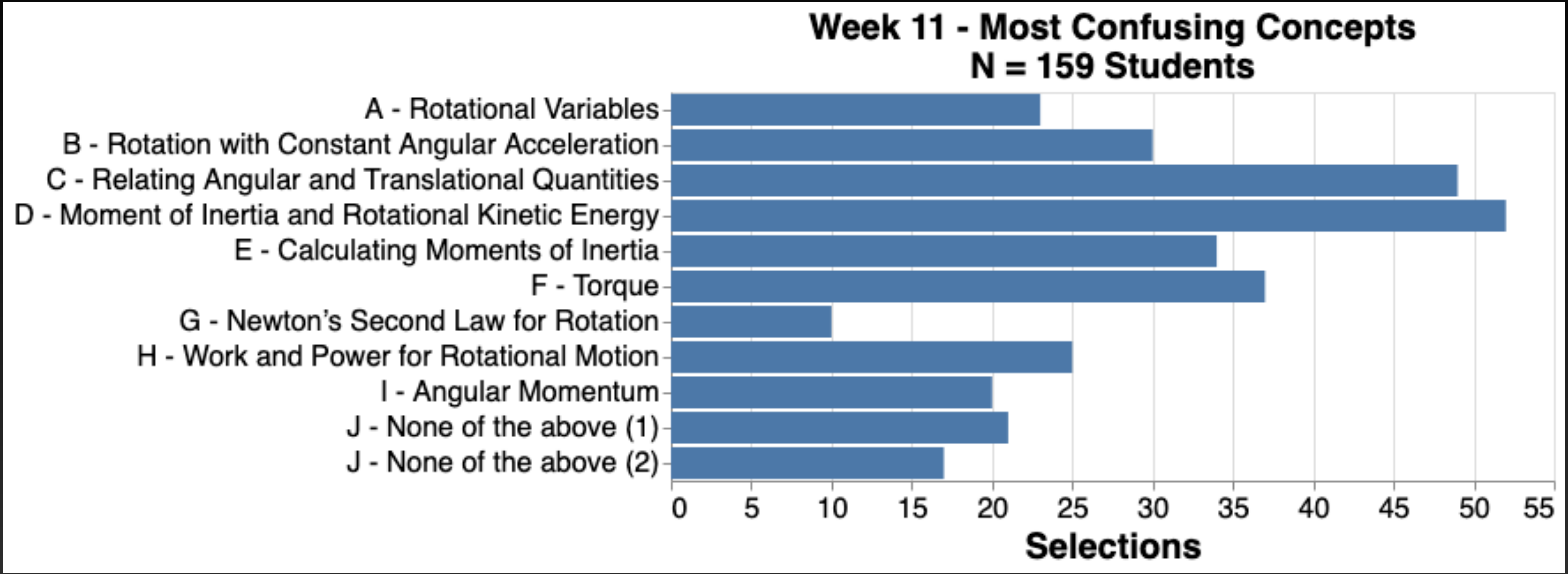
Chapter Outline

In previous chapters, we described motion (kinematics) and how to change motion (dynamics), and we defined important concepts such as energy for objects that can be considered as point masses. Point masses, by definition, have no shape and so can only undergo translational motion. However, we know from everyday life that rotational motion is also very important and that many objects that move have both translation and rotation. The wind turbines in our chapter opening image are a prime example of how rotational motion impacts our daily lives, as the market for clean energy sources continues to grow.

Friday's Class

10.6 Torque
10.7 Newton's Second Law for Rotation
10.8 Work and Power for Rotational Motion

HW 10 Reflection



Most confusing concepts:

What IS a "moment of inertia" ?

Things moving in circles is confusing

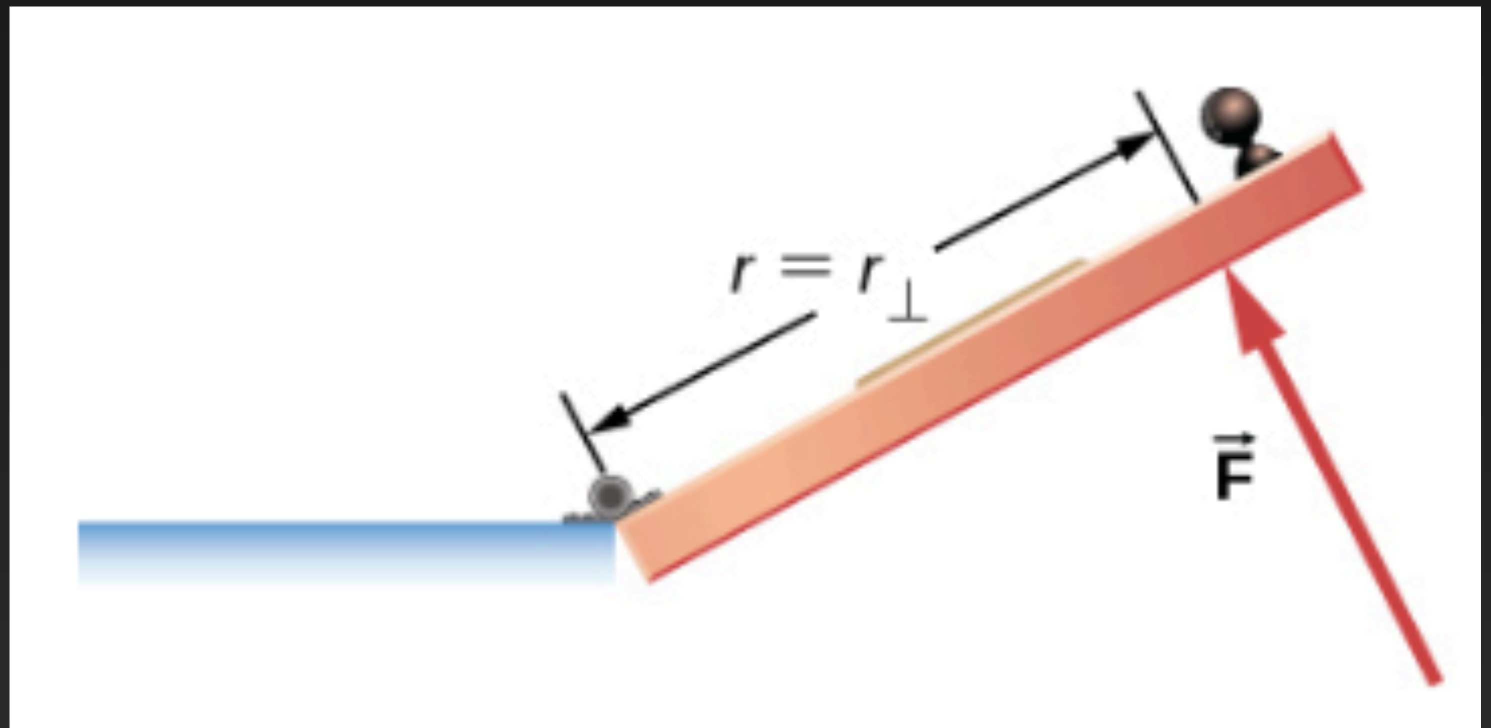
Torque is new and scary...

So many EQUATIONS!

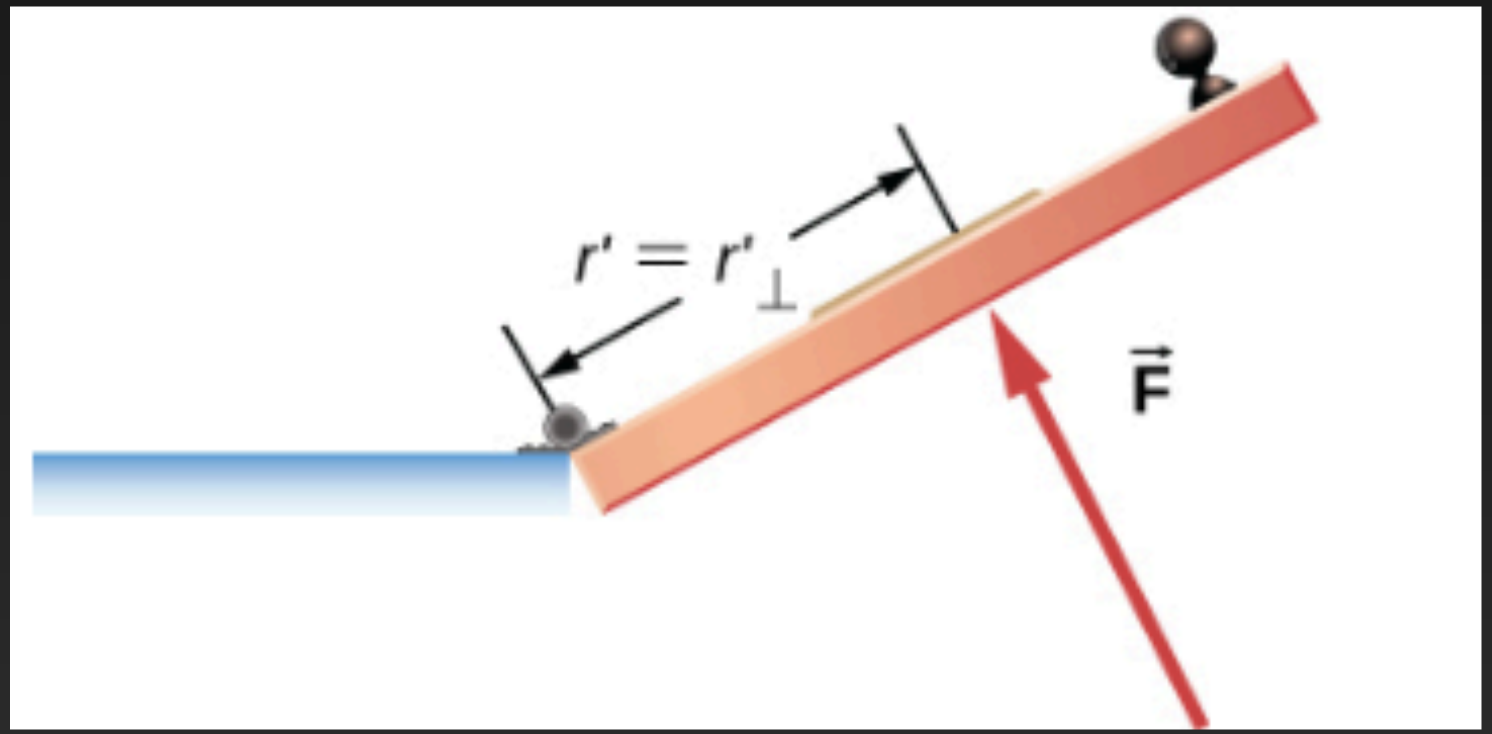
Rotational analogue for Force

A force F is applied to three different points on this door and hinge (looked at from above).

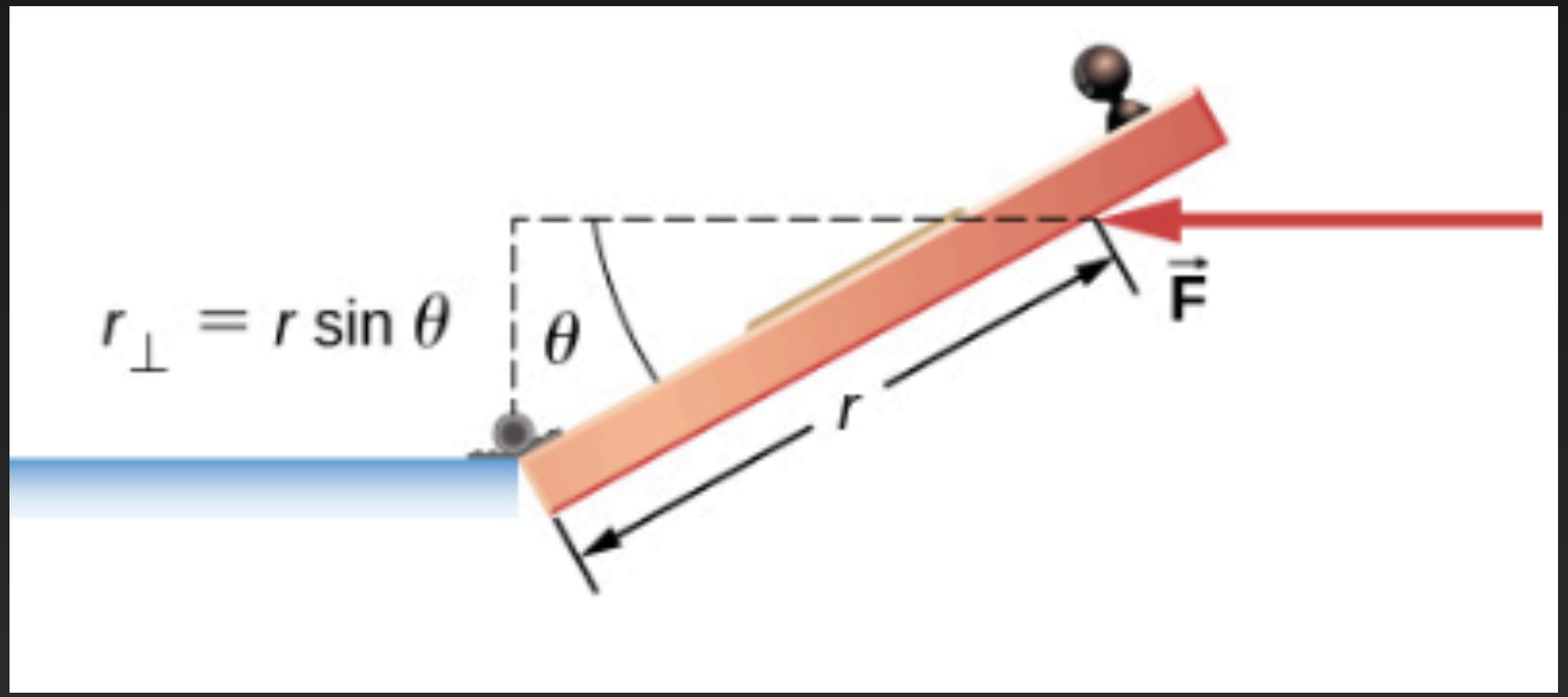
Which case will make the door open faster?



A) Far from hinge, force applied perpendicular to the door.



B) Closer to hinge, force applied perpendicular to the door



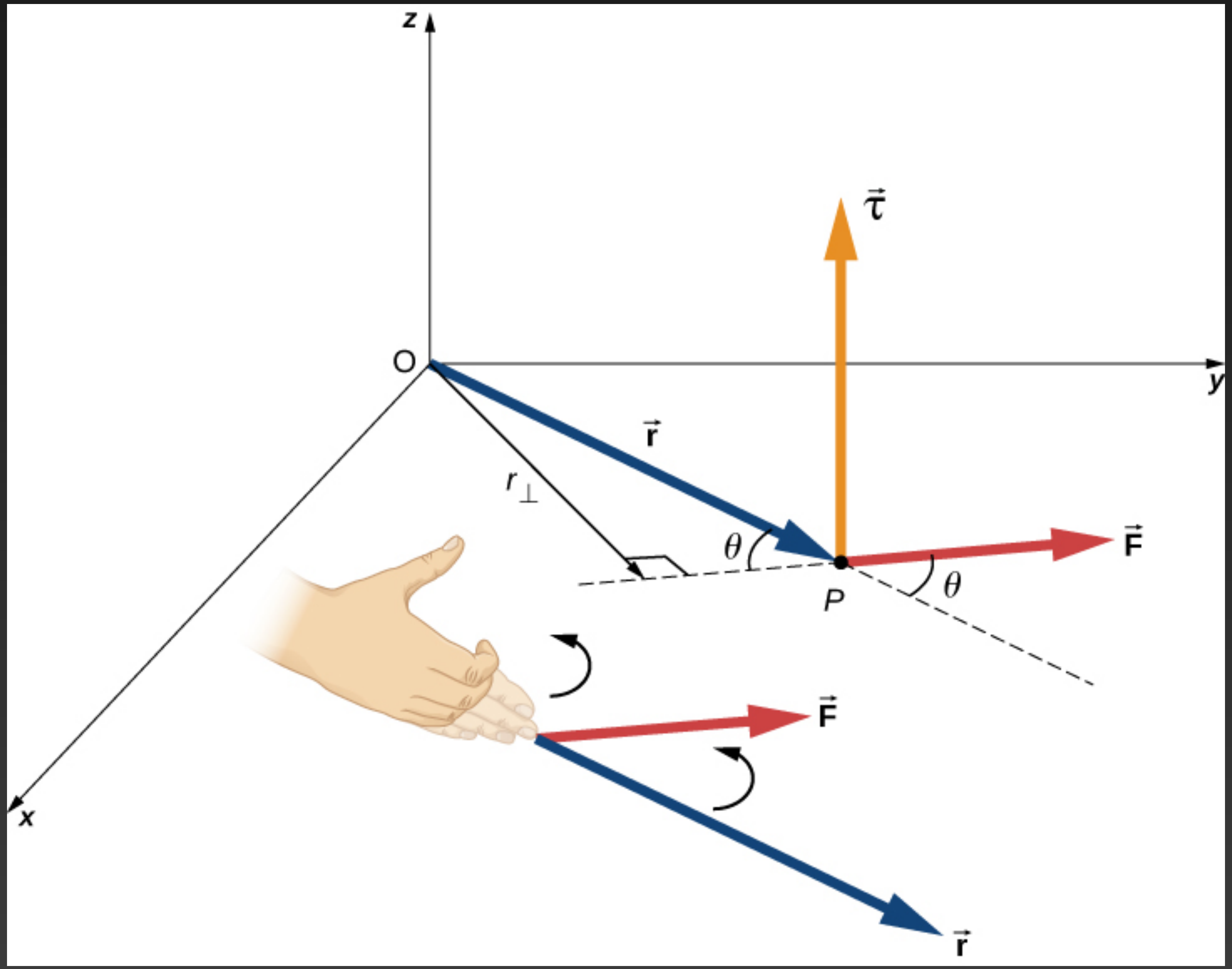
C) Far from hinge, force applied per

Torque

TORQUE

When a force $\vec{\mathbf{F}}$ is applied to a point P whose position is $\vec{\mathbf{r}}$ relative to O (Figure 10.32), the torque $\vec{\tau}$ around O is

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}. \tag{10.22}$$



Newton's second law for Rotation

NEWTON'S SECOND LAW FOR ROTATION

If more than one torque acts on a rigid body about a fixed axis, then the sum of the torques equals the moment of inertia times the angular acceleration:

$$\sum_i \tau_i = I\alpha. \tag{10.25}$$

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$$\sum_i \tau_i = I\alpha. \qquad (10.25)$$

Remember:

NEWTON'S SECOND LAW OF MOTION

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system and is inversely proportion to its mass. In equation form, Newton's second law is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{F}}_{\text{net}}}{m},$$

where $\vec{\mathbf{a}}$ is the acceleration, $\vec{\mathbf{F}}_{\text{net}}$ is the net force, and m is the mass. This is often written in the more familiar form

$$\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}, \qquad (5.3)$$

but the first equation gives more insight into what Newton's second law means. When only the magnitude of force and acceleration are considered, this equation can be written in the simpler scalar form:

$$F_{\text{net}} = ma. \qquad (5.4)$$

Torque

Work in Rotational Motion

WORK-ENERGY THEOREM FOR ROTATION

The work-energy theorem for a rigid body rotating around a fixed axis is

$$W_{AB} = K_B - K_A \tag{10.29}$$

where

$$K = \frac{1}{2} I \omega^2$$

and the rotational work done by a net force rotating a body from point *A* to point *B* is

$$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta. \tag{10.30}$$

Example

EXAMPLE 10.17

Rotational Work and Energy

A $12.0\,\mathrm{N \cdot m}$ torque is applied to a flywheel that rotates about a fixed axis and has a moment of inertia of $30.0\,\mathrm{kg \cdot m^2}$. If the flywheel is initially at rest, what is its angular velocity after it has turned through eight revolutions?

Strategy

We apply the work-energy theorem. We know from the problem description what the torque is and the angular displacement of the flywheel. Then we can solve for the final angular velocity.

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Strategy

We apply the work-energy theorem. We know from the problem description what the torque is and the angular displacement of the flywheel. Then we can solve for the final angular velocity.

Solution

The flywheel turns through eight revolutions, which is 16π radians. The work done by the torque, which is constant and therefore can come outside the integral in Equation 10.30, is

$$W_{AB} = \tau(\theta_B - \theta_A).$$

We apply the work-energy theorem:

$$W_{AB} = \tau(\theta_B - \theta_A) = \frac{1}{2}I\omega_B^2 - \frac{1}{2}I\omega_A^2.$$

With $\tau = 12.0\,\text{N}\cdot\text{m}$, $\theta_B - \theta_A = 16.0\pi\,\text{rad}$, $I = 30.0\,\text{kg}\cdot\text{m}^2$, and $\omega_A = 0$, we have

$$12.0\,\text{N-m}(16.0\pi\,\text{rad}) = \frac{1}{2}(30.0\,\text{kg}\cdot\text{m}^2)(\omega_B^2) - 0.$$

Therefore,

$$\omega_B = 6.3\,\text{rad/s}.$$

This is the angular velocity of the flywheel after eight revolutions.

Significance

The work-energy theorem provides an efficient way to analyze rotational motion, connecting torque with rotational kinetic energy.

Power in Rotational Motion

$$P = \tau\omega. \quad (10.31)$$

to rotational motion. From Work and Kinetic Energy, the instantaneous power (or just power) is defined as the rate of doing work,

$$P = \frac{dW}{dt}.$$

If we have a constant net torque, Equation 10.25 becomes $W = \tau\theta$ and the power is

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\frac{d\theta}{dt}$$

Rotational Analogues

Rotational	Translational
	$\sum_i \vec{\mathbf{F}}_i = m\vec{\mathbf{a}}$
	$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$
	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

Rotational Analogues

Rotational	Translational
$\sum_i \tau_i = I\alpha$	$\sum_i \vec{\mathbf{F}}_i = m\vec{\mathbf{a}}$
$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta$	$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$
$P = \tau\omega$	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

Key Equations

Angular position	$\theta = \frac{s}{r}$
Angular velocity	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
Tangential speed	$v_t = r\omega$
Angular acceleration	$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Tangential acceleration	$a_t = r\alpha$
Average angular velocity	$\bar{\omega} = \frac{\omega_0 + \omega_f}{2}$
Angular displacement	$\theta_f = \theta_0 + \bar{\omega} t$
Angular velocity from constant angular acceleration	$\omega_f = \omega_0 + \alpha t$
Angular velocity from displacement and constant angular acceleration	$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
Change in angular velocity	$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$
Total acceleration	$\vec{\mathbf{a}} = \vec{\mathbf{a}}_c + \vec{\mathbf{a}}_t$

Key Equations

Rotational kinetic energy	$K = \frac{1}{2}\left(\sum_j m_j r_j^2\right)\omega^2$
Moment of inertia	$I = \sum_j m_j r_j^2$
Rotational kinetic energy in terms of the moment of inertia of a rigid body	$K = \frac{1}{2} I \omega^2$
Moment of inertia of a continuous object	$I = \int r^2 dm$
Parallel-axis theorem	$I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2$
Moment of inertia of a compound object	$I_{\text{total}} = \sum_i I_i$

Key Equations

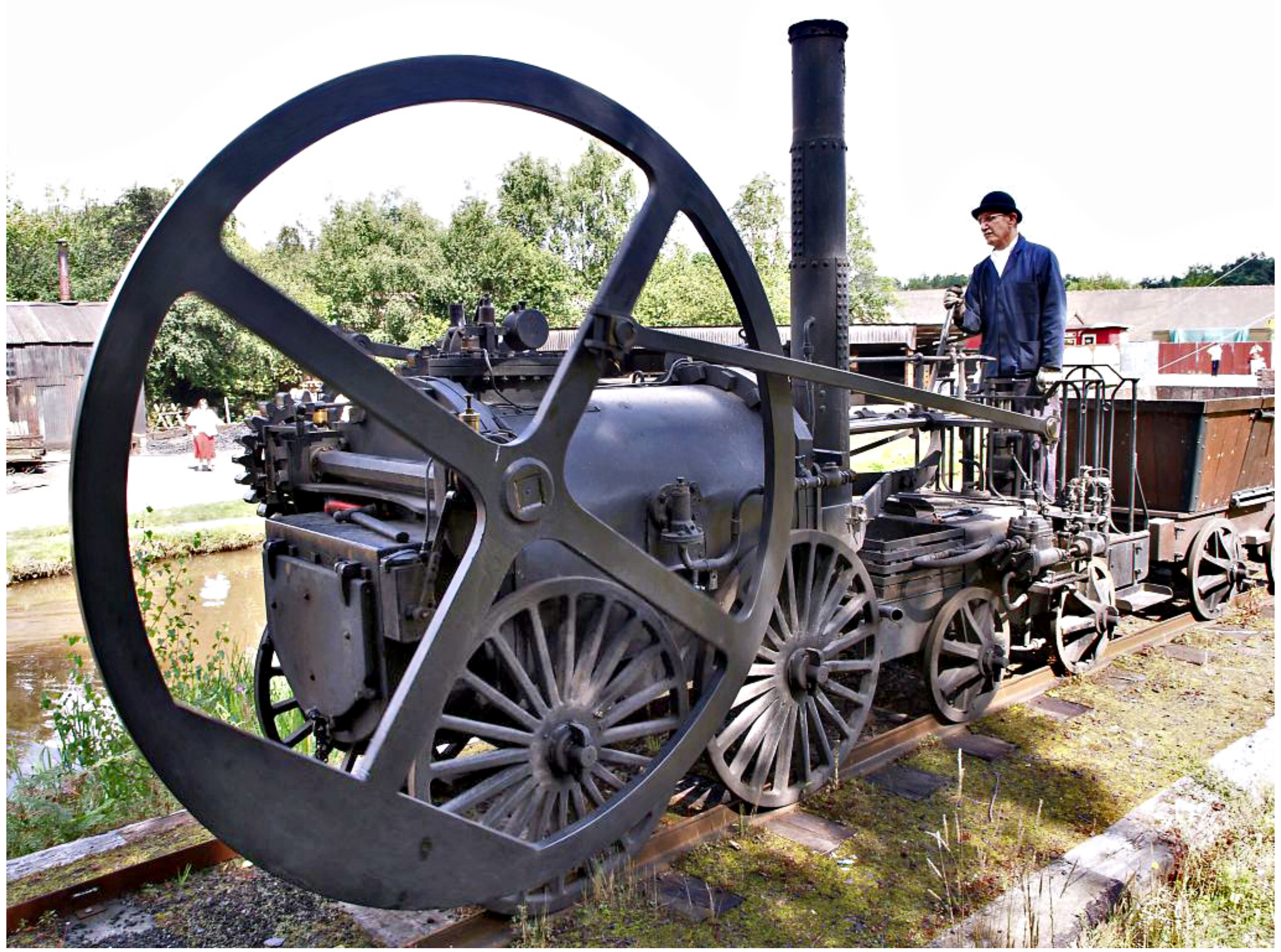
Torque vector	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Magnitude of torque	$\left\lvert\vec{\boldsymbol{\tau}}\right\rvert = r_{\perp} F$
Total torque	$\vec{\tau}_{\text{net}} = \sum_i \lvert\vec{\tau}_i\rvert$
Newton's second law for rotation	$\sum_i \tau_i = I\alpha$
Incremental work done by a torque	$dW = \left(\sum_i \tau_i\right) d\theta$
Work-energy theorem	$W_{AB} = K_B - K_A$
Rotational work done by net force	$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i\right) d\theta$
Rotational power	$P = \tau\omega$

Activity:
Worked Problems

Rotational Work and Energy

A $12.0\,\mathrm{N}\cdot\mathrm{m}$ torque is applied to a flywheel that rotates about a fixed axis and has a moment of inertia of $30.0\,\mathrm{kg}\cdot\mathrm{m}^2$. If the flywheel is initially at rest, what is its angular velocity after it has turned through eight revolutions?

WP 12.1



See you next class!

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