You can draw here

Physics 111 - Class 12B
Fixed Axis Rotation

Do not draw in/on this box!

November 24, 2021

You can draw here

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Class Outline

- Logistics / Announcements
- Chapter 10 Section Summary
  - Lots of talking from me today, SORRY!

Logistics/Announcements

- Lab this week: Lab 8
- HW10 due this week on Thursday at 6 PM
- Learning Log 10 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Test 5 available this week (Chapters 8 & 9)
    - Test Window: Friday 6 PM - Sunday 6 PM
    - Last one!

Physics 111

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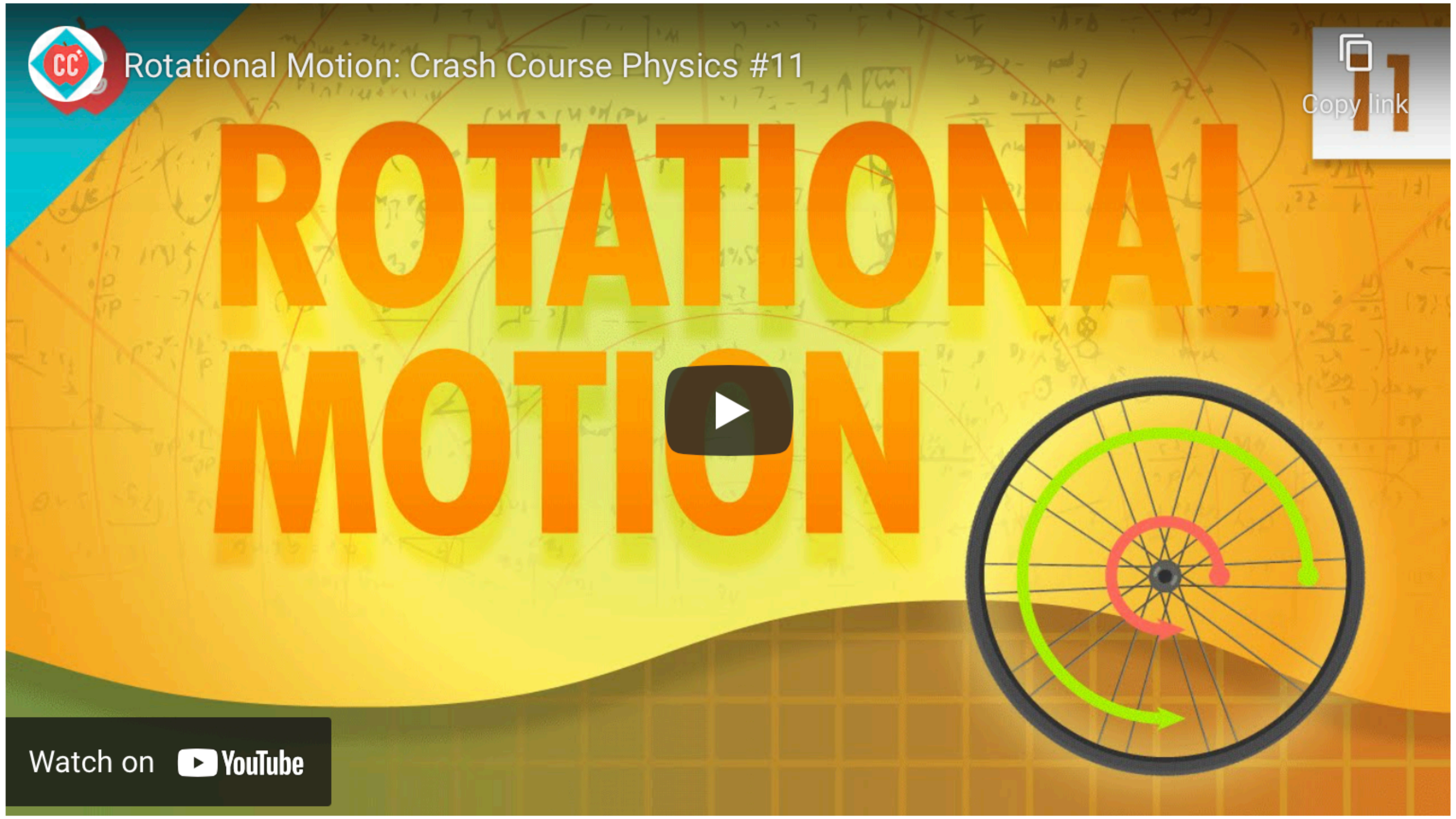
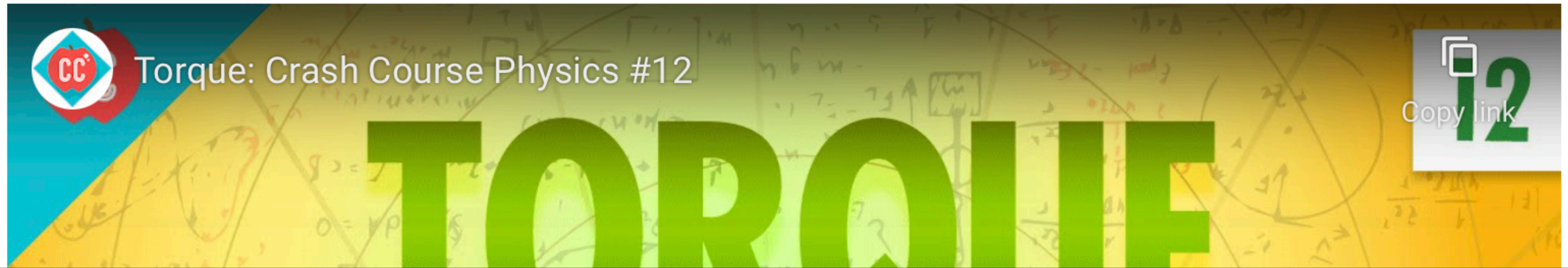
Contents

Rotational Motion



Torque



Wed

Fri

Figure 10.1 Brazos wind farm in west Texas. During 2019, wind farms in the United States had an average power output of 34 gigawatts, which is enough to power 28 million homes. (credit: modification of work by U.S. Department of Energy)

Chapter Outline

In previous chapters, we described motion (kinematics) and how to change motion (dynamics), and we defined important concepts such as energy for objects that can be considered as point masses. Point masses, by definition, have no shape and so can only undergo translational motion. However, we know from everyday life that rotational motion is also very important and that many objects that move have both translation and rotation. The wind turbines in our chapter opening image are a prime example of how rotational motion impacts our daily lives, as the market for clean energy sources continues to grow.

Wednesday’s Class

10.1 Rotational Variables
10.2 Rotation with Constant Angular Acceleration
10.3 Relating Angular and Translational Quantities
10.4 Moment of Inertia and Rotational Kinetic Energy
10.5 Calculating Moments of Inertia

Rotational Variables

- So far in this course we have mostly done "translational motion" in x, y, or z
    - Quantities: Displacement, velocity, and acceleration
- As we become more sophisticated physicists, we realize that we have ignored "**rotational motion**"
    - Quantities: Angular displacement, Angular velocity, Angular acceleration

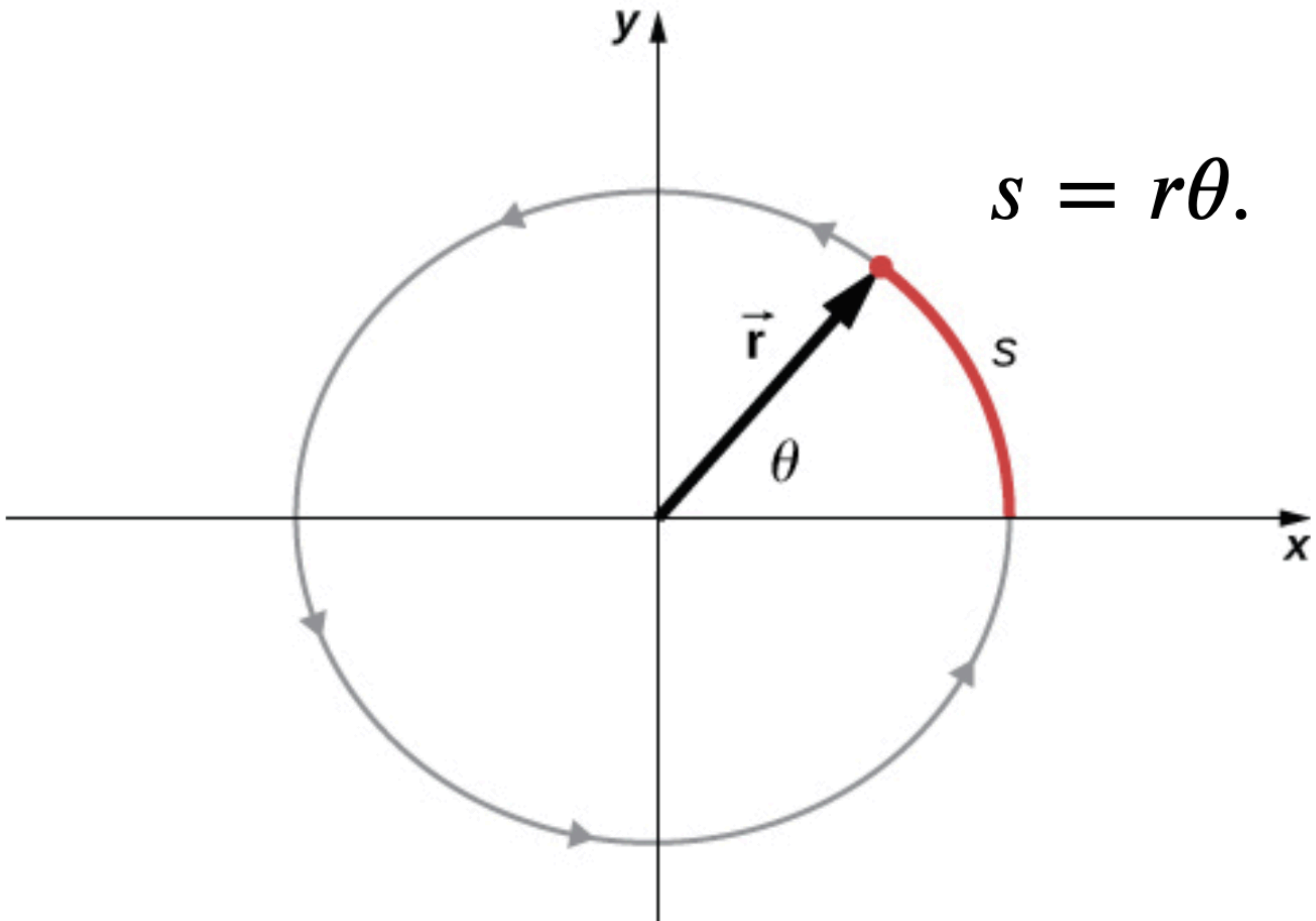
Rotational Quantities

Rotational	Translational	Relationship (r = radius)
θ	s	$\theta = \frac{s}{r}$
ω	v_t	$\omega = \frac{v_t}{r}$
α	a_t	$\alpha = \frac{a_t}{r}$
	a_c	$a_c = \frac{v_t^2}{r}$

Table 10.3 Rotational and Translational Quantities: Circular Motion

Rotational Motion

In Figure 10.2, we show a particle moving in a circle. The coordinate system is fixed and serves as a frame of reference to define the particle's position. Its position vector from the origin of the circle to the particle sweeps out the angle θ, which increases in the counterclockwise direction as the particle moves along its circular path. The angle θ is called the **angular position** of the particle. As the particle moves in its circular path, it also traces an arc length s.



$$s = r\theta.$$

Rotational Motion

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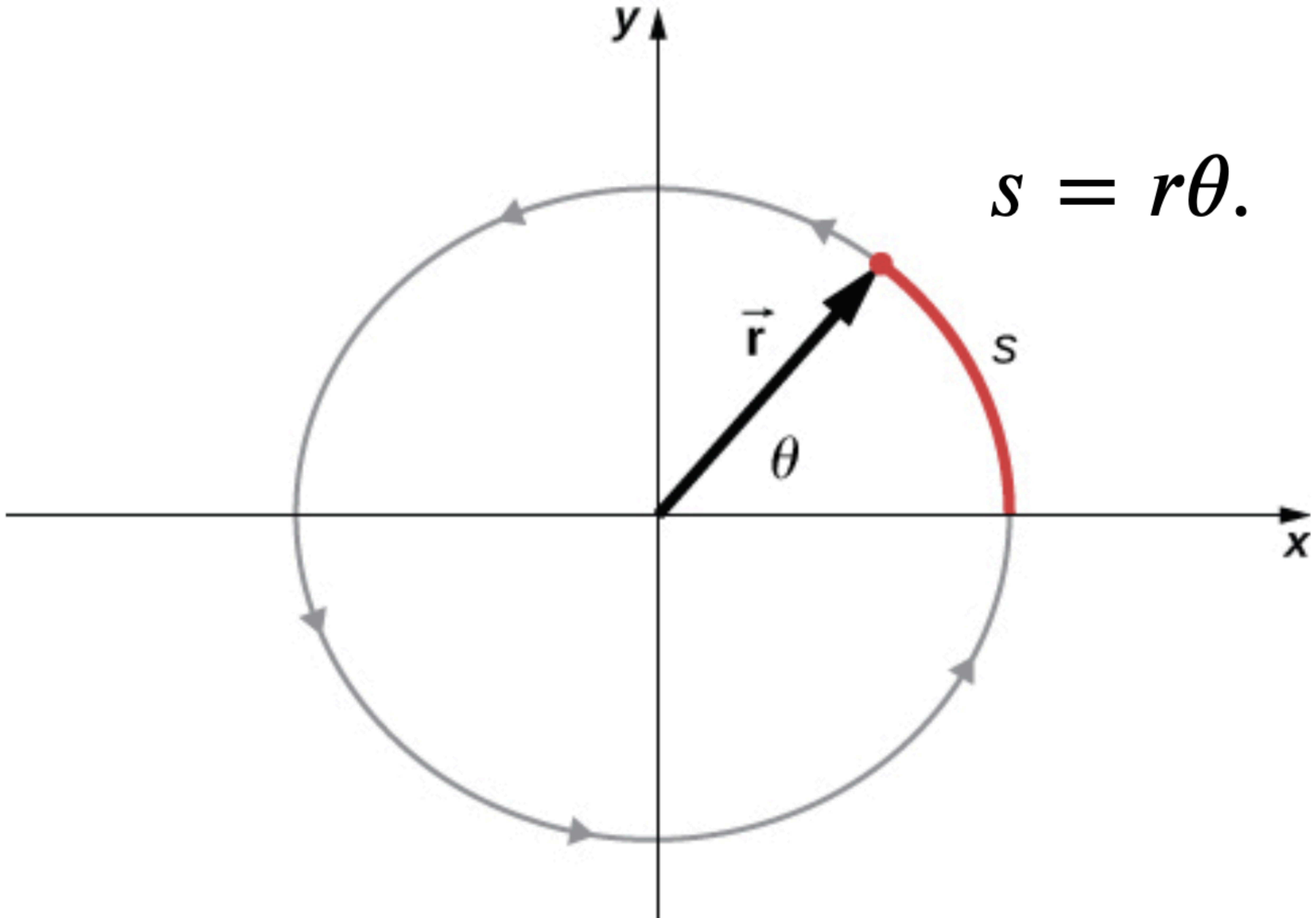

$$s = r\theta.$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt},$$

Rotational Motion

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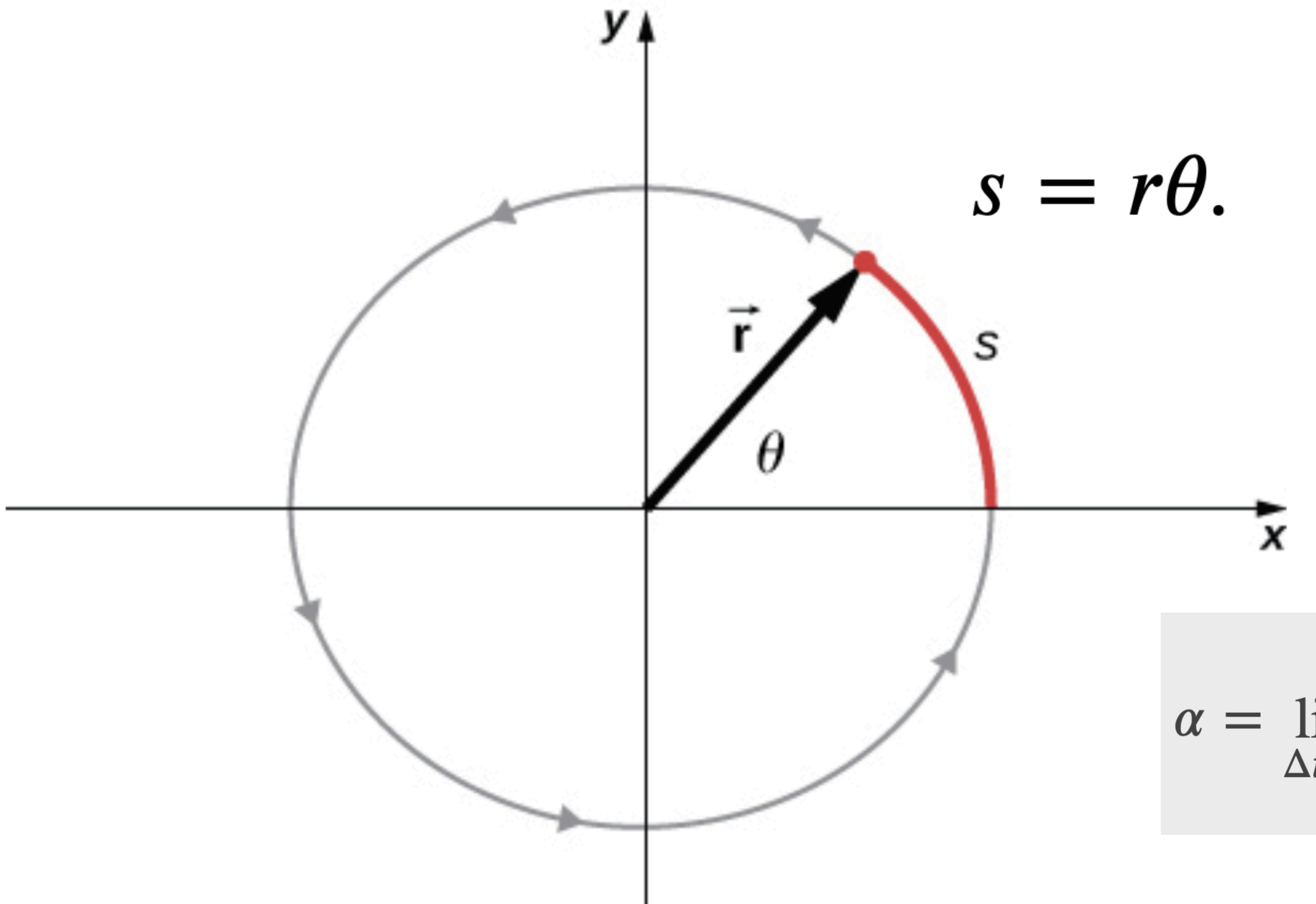


$$s = r\theta.$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt},$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2},$$

Rotating Disk

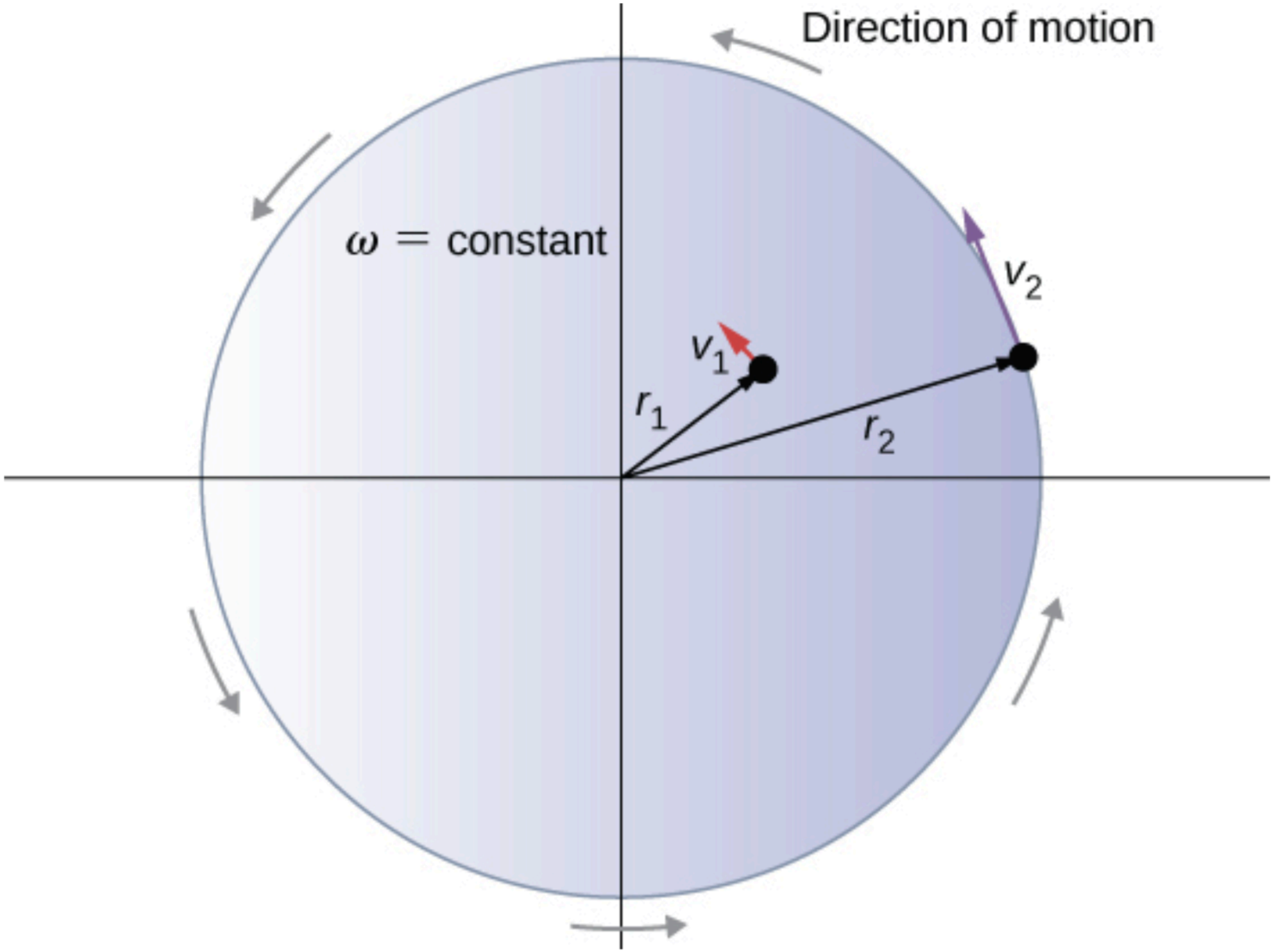
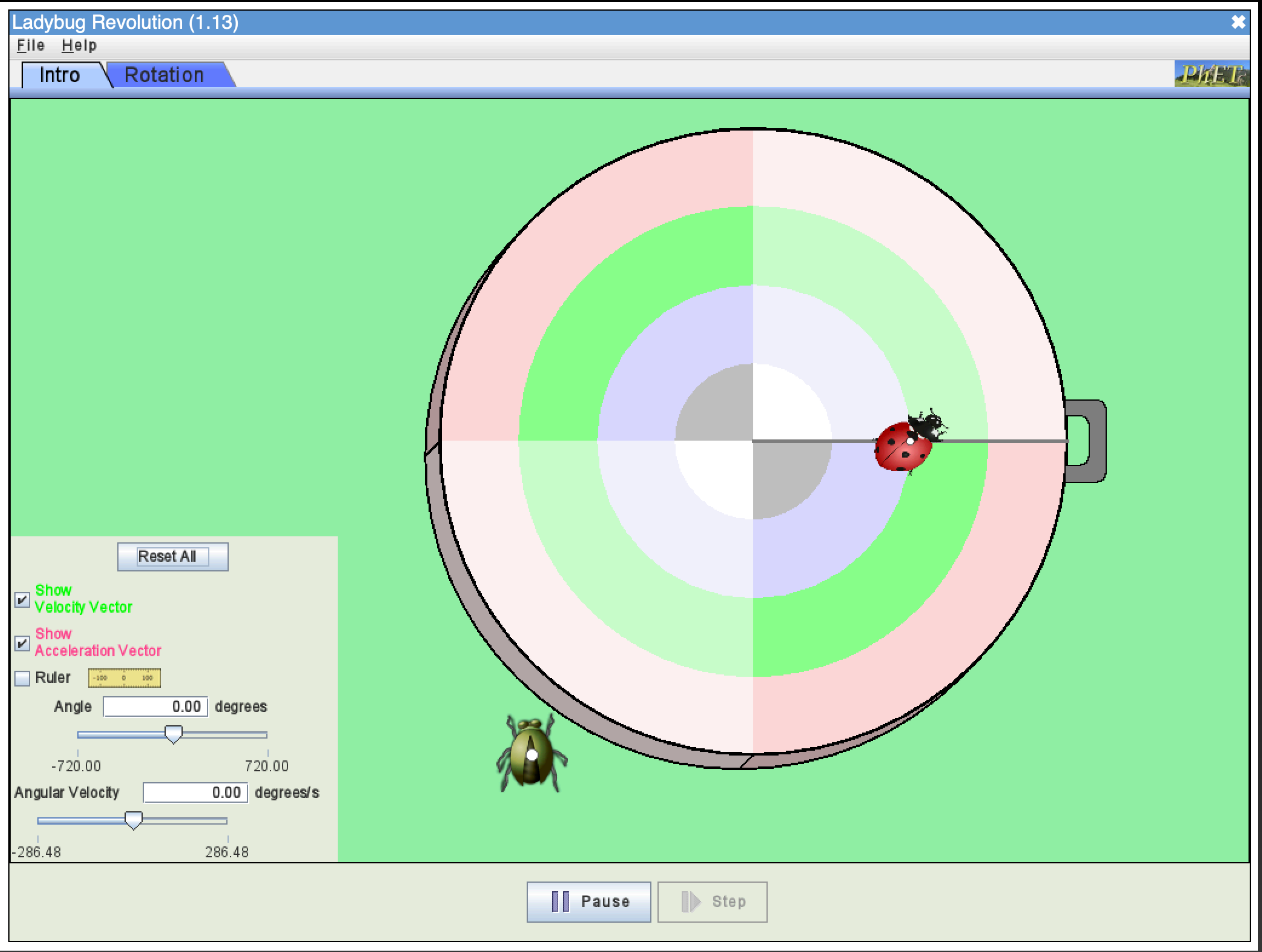


Figure 10.4 Two particles on a rotating disk have different tangential speeds, depending on their distance to the axis of rotation.

Rotating Disk



Link to PHET Simulation: https://phet.colorado.edu/sims/cheerpj/rotation/latest/rotation.html?simulation=rotation

Rotational Analogues

Translational
$x = x_0 + \bar{v}t$
$v_f = v_0 + at$
$x_f = x_0 + v_0 t + \frac{1}{2}at^2$
$v_f^2 = v_0^2 + 2a(\Delta x)$

Table 10.2 Rotational and Translational Kinematic Equations

Rotational Analogues

Rotational	Translational
$\theta_f = \theta_0 + \bar{\omega} t$	$x = x_0 + \bar{v} t$
$\omega_f = \omega_0 + \alpha t$	$v_f = v_0 + a t$
$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$	$x_f = x_0 + v_0 t + \frac{1}{2} a t^2$
$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$	$v_f^2 = v_0^2 + 2a(\Delta x)$

Table 10.2 Rotational and Translational Kinematic Equations

Rotational Analogues

Translational
m
$K = \frac{1}{2}mv^2$

Table 10.4 Rotational and Translational Kinetic Energies and Inertia

Rotational Analogues

Rotational	Translational
$I = \sum_j m_j r_j^2$	m
$K = \frac{1}{2} I \omega^2$	$K = \frac{1}{2} m v^2$

Table 10.4 Rotational and Translational Kinetic Energies and Inertia

Rotational Inertia

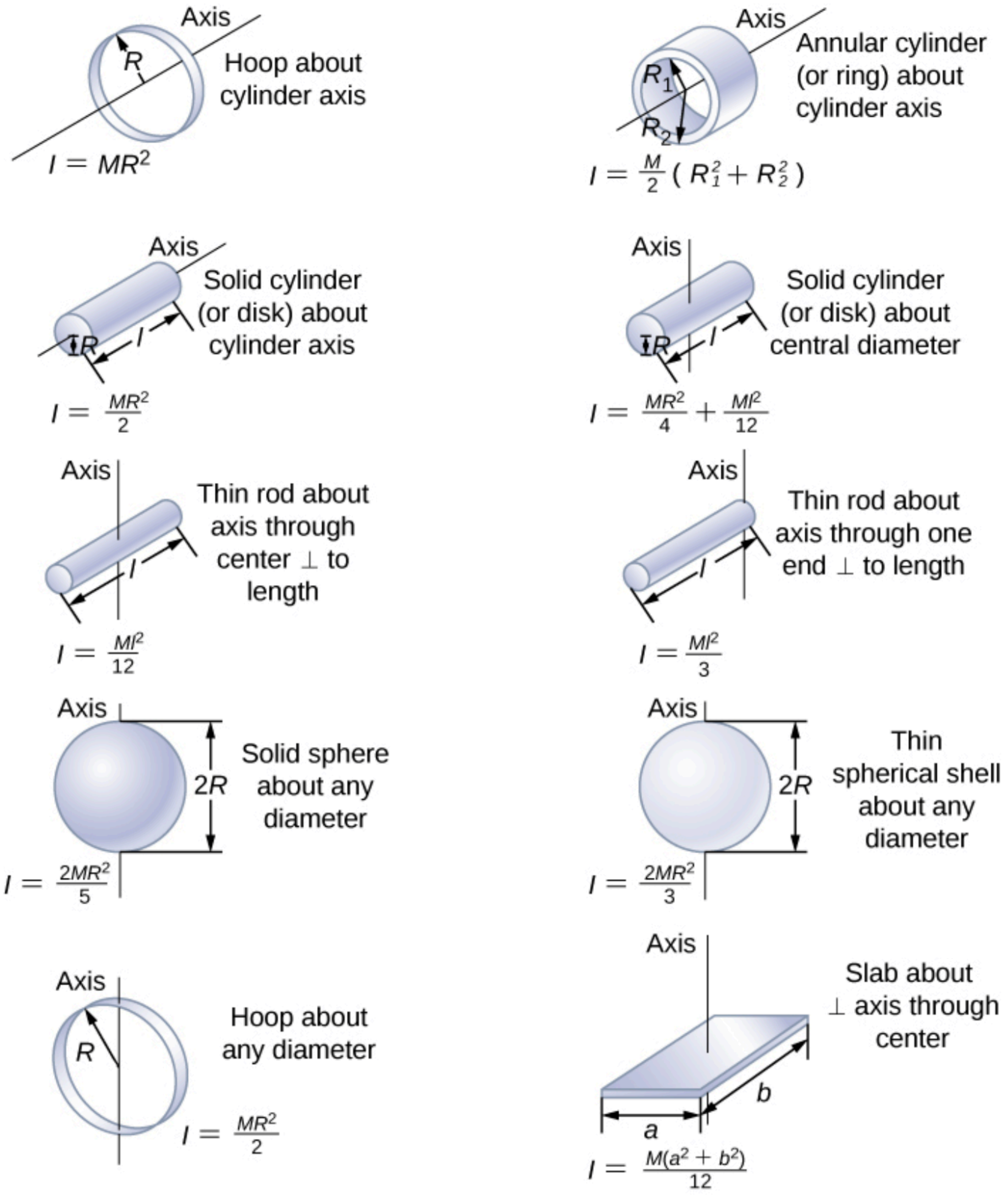
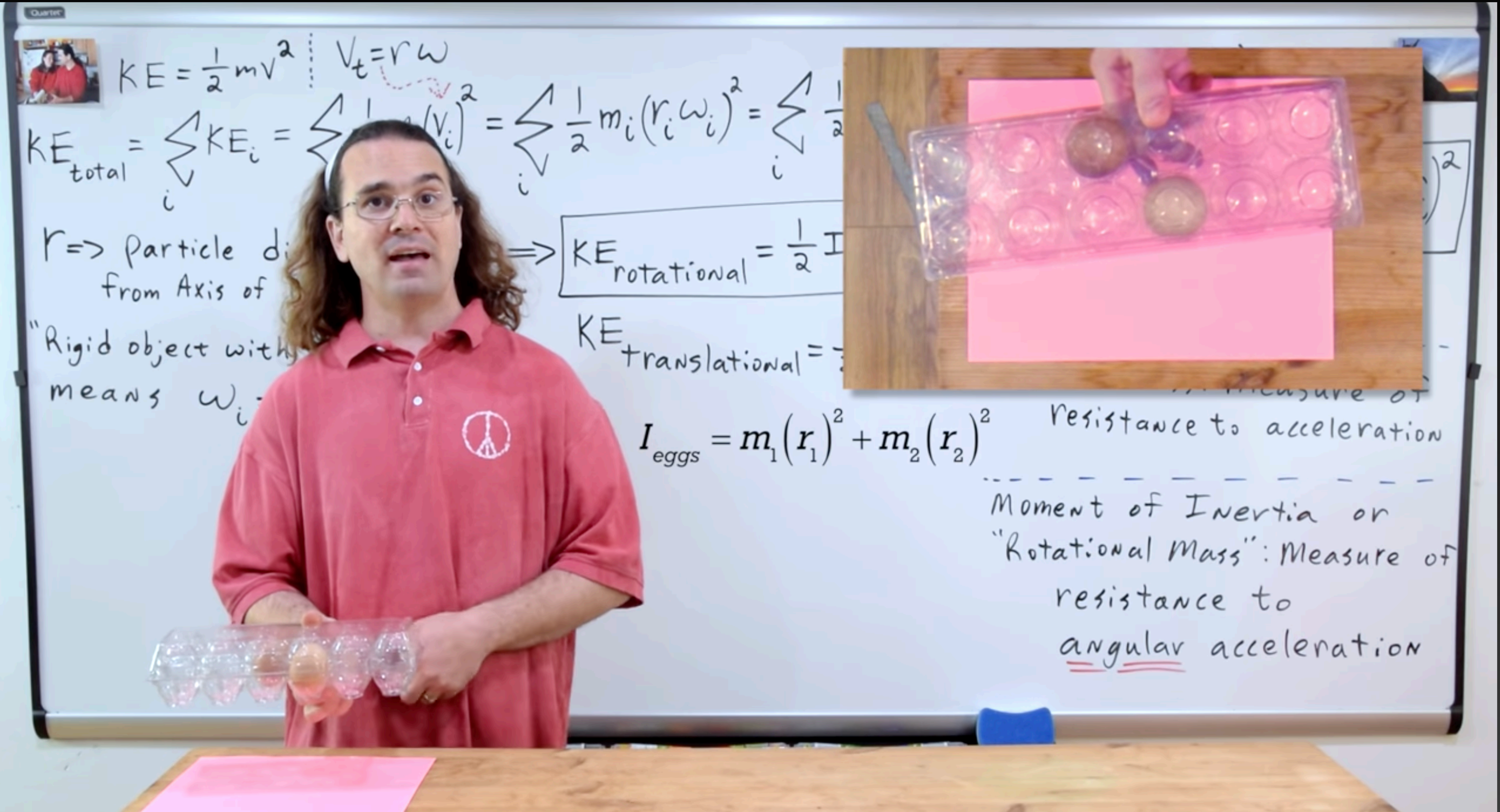


Figure 10.20 Values of rotational inertia for common shapes of objects.

Deriving Rotational Kinetic Energy



Example: Sphere rolling down a ramp

A ball (solid sphere) of mass m and radius R, rolls down a ramp without slipping. What is its velocity at the bottom of the ramp?

Example: Sphere rolling down a ramp

A ball (solid sphere) of mass m and radius R, rolls down a ramp without slipping. What is its velocity at the bottom of the ramp?

$$v = \sqrt{\frac{10}{7} gh}$$

Key Equations

Angular position	$\theta = \frac{s}{r}$
Angular velocity	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
Tangential speed	$v_{\mathrm{t}} = r\omega$
Angular acceleration	$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
Tangential acceleration	$a_{\mathrm{t}} = r\alpha$
Average angular velocity	$\bar{\omega} = \frac{\omega_0 + \omega_{\mathrm{f}}}{2}$
Angular displacement	$\theta_{\mathrm{f}} = \theta_0 + \bar{\omega} t$
Angular velocity from constant angular acceleration	$\omega_{\mathrm{f}} = \omega_0 + \alpha t$
Angular velocity from displacement and constant angular acceleration	$\theta_{\mathrm{f}} = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
Change in angular velocity	$\omega_{\mathrm{f}}^2 = \omega_0^2 + 2\alpha(\Delta\theta)$
Total acceleration	$\vec{\mathbf{a}} = \vec{\mathbf{a}}_{\mathrm{c}} + \vec{\mathbf{a}}_{\mathrm{t}}$

Key Equations

Rotational kinetic energy	$K = \frac{1}{2}\left(\sum_j m_j r_j^2\right)\omega^2$
Moment of inertia	$I = \sum_j m_j r_j^2$
Rotational kinetic energy in terms of the moment of inertia of a rigid body	$K = \frac{1}{2} I \omega^2$
Moment of inertia of a continuous object	$I = \int r^2 dm$
Parallel-axis theorem	$I_{\text{parallel-axis}} = I_{\text{center of mass}} + md^2$
Moment of inertia of a compound object	$I_{\text{total}} = \sum_i I_i$

Key Equations

Torque vector	$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$
Magnitude of torque	$\lvert\vec{\boldsymbol{\tau}}\rvert = r_{\perp} F$
Total torque	$\vec{\tau}_{\text{net}} = \sum_i \lvert\vec{\tau}_i\rvert$
Newton's second law for rotation	$\sum_i \tau_i = I\alpha$
Incremental work done by a torque	$dW = \left(\sum_i \tau_i\right) d\theta$
Work-energy theorem	$W_{AB} = K_B - K_A$
Rotational work done by net force	$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i\right) d\theta$
Rotational power	$P = \tau\omega$

See you next class!

Attribution