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Physics 111 - Class 11C
Momentum & Impulse

November 19, 2021

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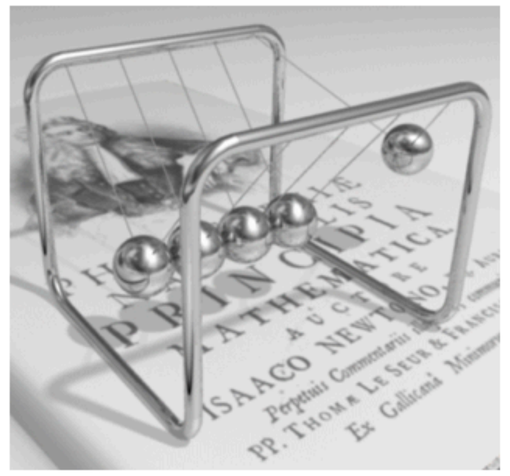
You can draw here

Class Outline

- Logistics / Announcements
- Homework Reflection
- Chapter 9 Section Summary
- Clicker Questions
- Worked Problems
- Revisiting Bullet Block vs. Spinning Bullet Block

Logistics/Announcements

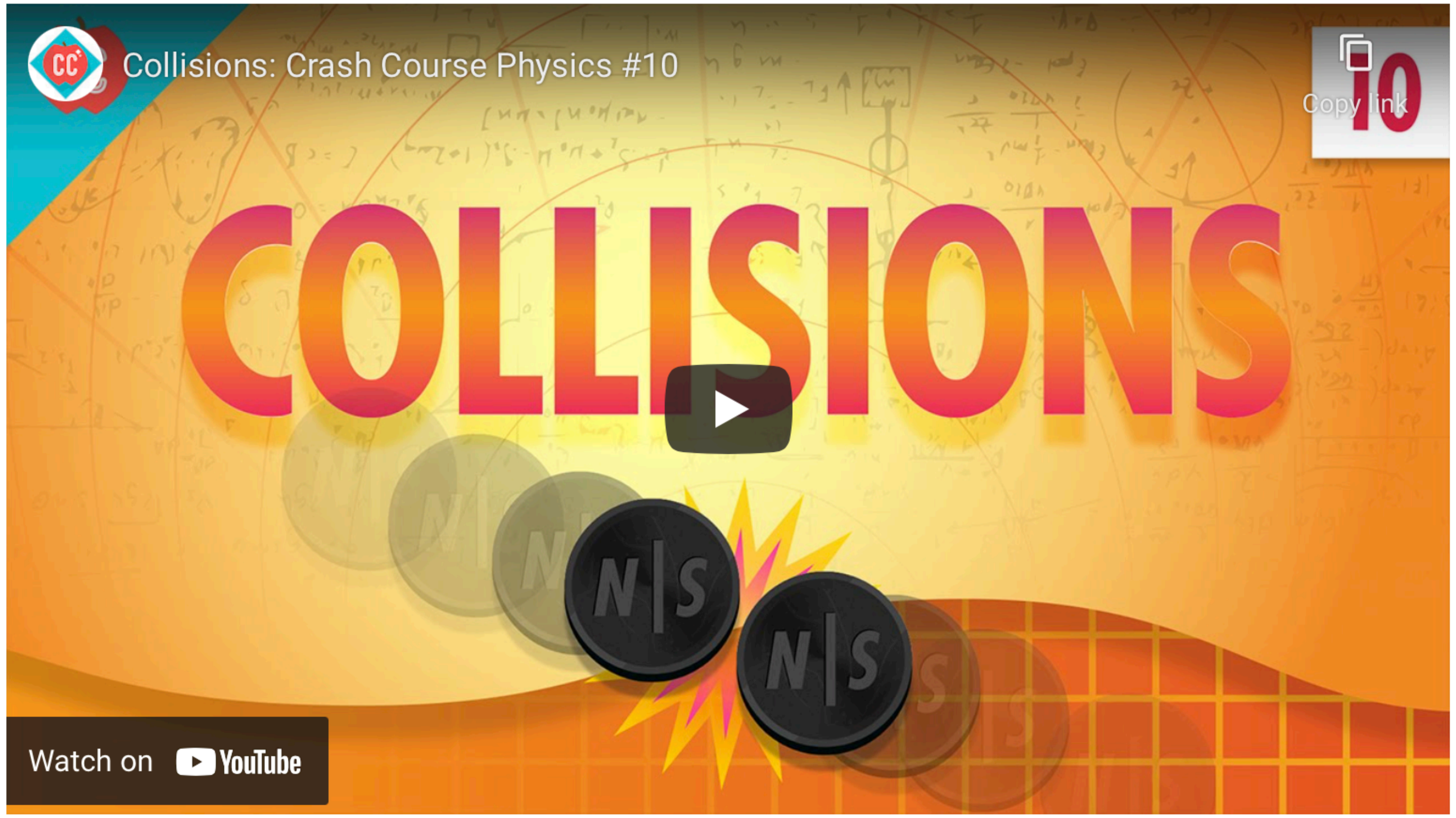
- Lab this week: Lab 7
- HW9 due this week on Thursday at 6 PM
- Learning Log 9 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Bonus Test 4 available this week (Chapters 7 & 8)
    - Test Window: Friday 6 PM - Sunday 6 PM



Physics 111

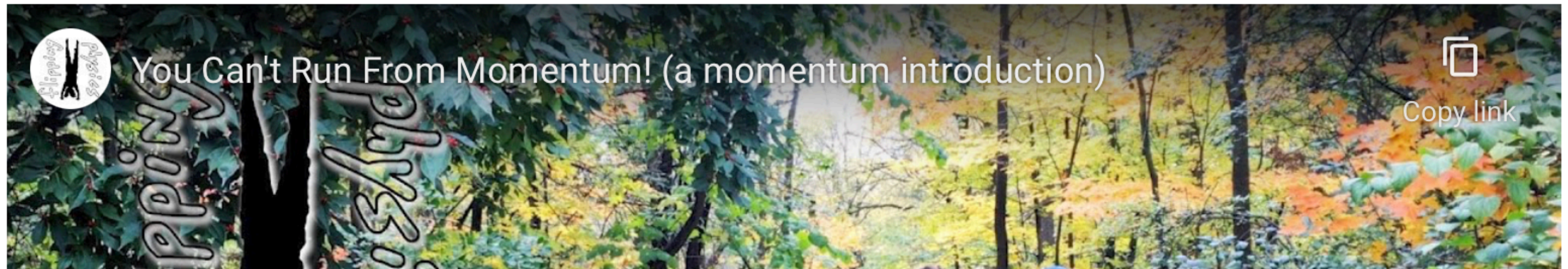
Content Summary from Crash Course Physics

Collisions



Required Videos

1. You Can't Run From Momentum! (a momentum introduction)



Checklist of items

- [ ] Video 1
- [ ] Video 2
- [ ] Video 3
- [ ] Video 4
- [ ] Video 5
- [ ] Video 6
- [ ] Video 7
- [ ] Video 8
- [ ] Video 9
- [ ] Video 10

Figure 9.1 The concepts of impulse, momentum, and center of mass are crucial for a major-league baseball player to successfully get a hit. If he misjudges these quantities, he might break his bat instead. (credit: modification of work by "Cathy T"/Flickr)

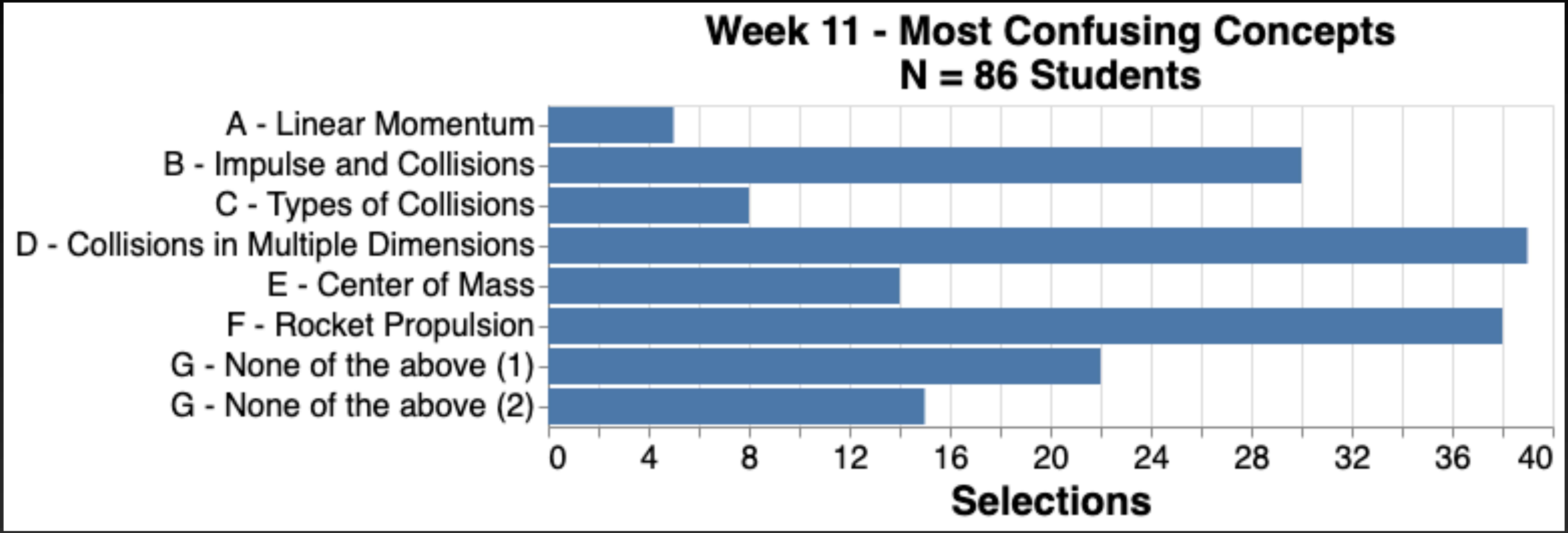
Chapter Outline

The concepts of work, energy, and the work-energy theorem are valuable for two primary reasons: First, they are powerful computational tools, making it much easier to analyze complex physical systems than is possible using

Friday Class

9.5 Collisions in multiple dimensions
9.7 Rocket Propulsion

HW 9 Reflection



Most confusing concepts:

Rocket Propulsion Equations

Collisions in Multiple Dimensions

Zero Momentum Frame
(relative velocities & Momentum)

Not bad!

Momentum in multiple dimensions

PROBLEM-SOLVING STRATEGY

Conservation of Momentum in Two Dimensions

The method for solving a two-dimensional (or even three-dimensional) conservation of momentum problem is generally the same as the method for solving a one-dimensional problem, except that you have to conserve momentum in both (or all three) dimensions simultaneously:

1. Identify a closed system.
2. Write down the equation that represents conservation of momentum in the *x*-direction, and solve it for the desired quantity. If you are calculating a vector quantity (velocity, usually), this will give you the *x*-component of the vector.
3. Write down the equation that represents conservation of momentum in the *y*-direction, and solve. This will give you the *y*-component of your vector quantity.
4. Assuming you are calculating a vector quantity, use the Pythagorean theorem to calculate its magnitude, using the results of steps 3 and 4.

Momentum in multiple dimensions

EXAMPLE 9.15

Exploding Scuba Tank

A common scuba tank is an aluminum cylinder that weighs 31.7 pounds empty (Figure 9.25). When full of compressed air, the internal pressure is between 2500 and 3000 psi (pounds per square inch). Suppose such a tank, which had been sitting motionless, suddenly explodes into three pieces. The first piece, weighing 10 pounds, shoots off horizontally at 235 miles per hour; the second piece (7 pounds) shoots off at 172 miles per hour, also in the horizontal plane, but at a $19°$ angle to the first piece. What is the mass and initial velocity of the third piece? (Do all work, and express your final answer, in SI units.)

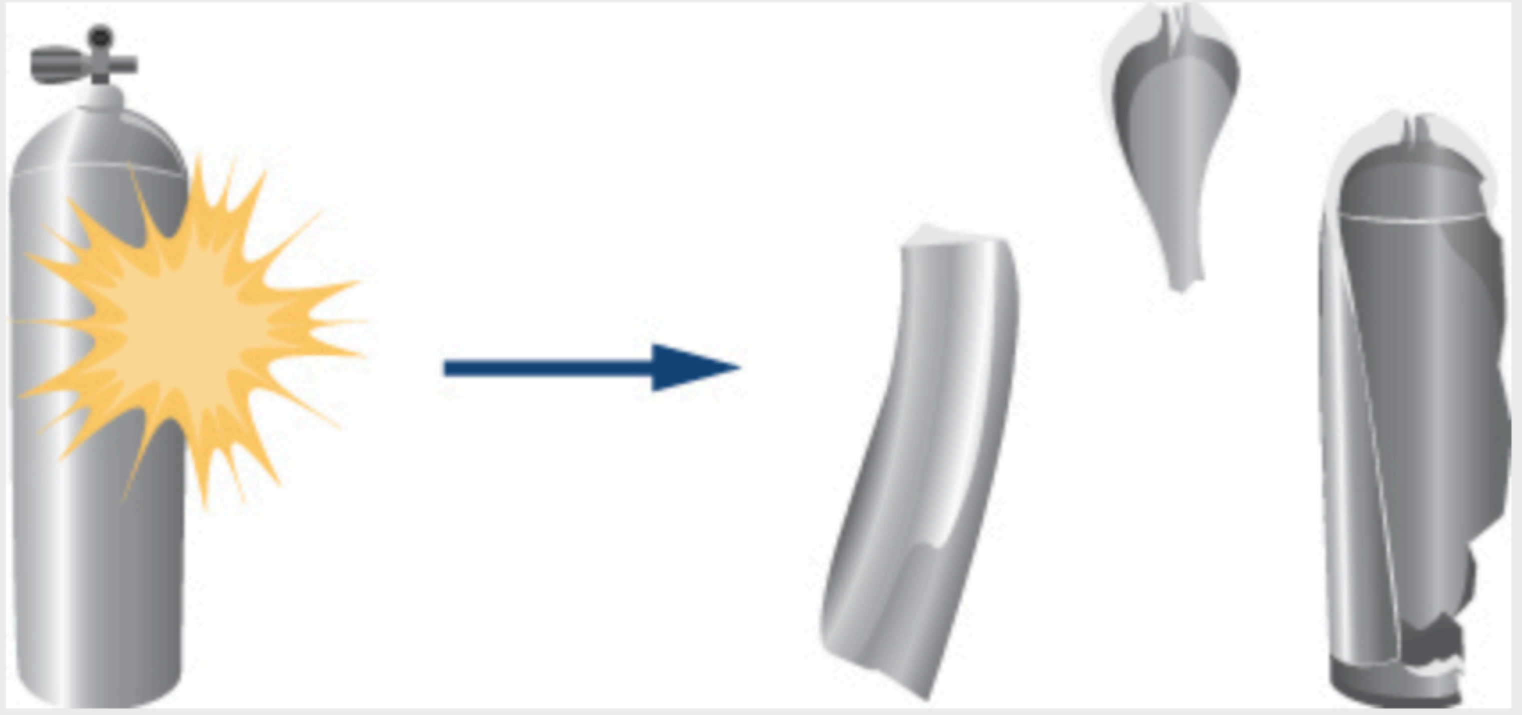


Figure 9.25 A scuba tank explodes into three pieces.

Momentum in multiple dimensions

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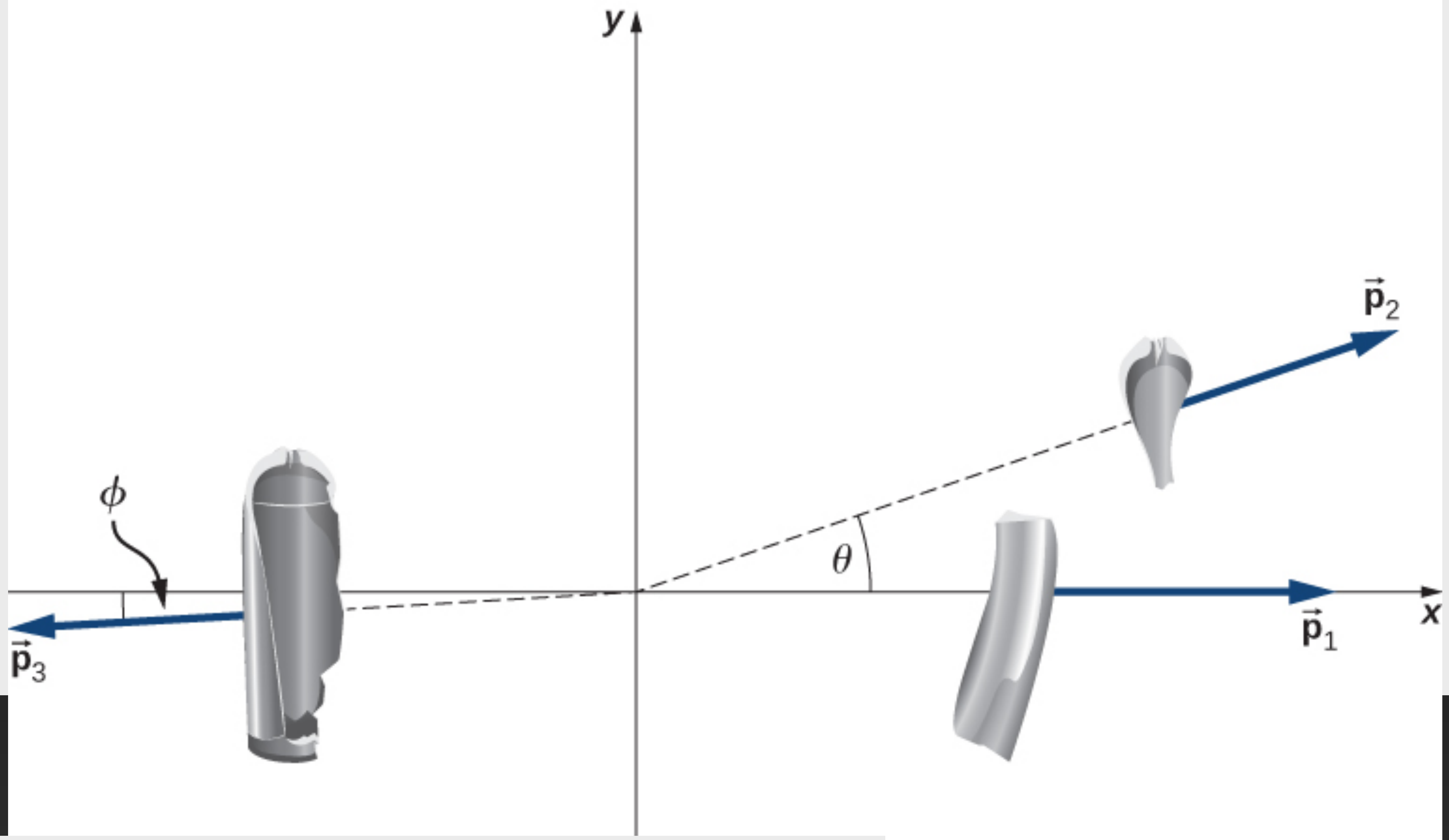

First, let's get all the conversions to SI units out of the way:

$$31.7\ \text{lb} \times \frac{1\ \text{kg}}{2.2\ \text{lb}} \rightarrow 14.4\ \text{kg}$$
$$10\ \text{lb} \rightarrow 4.5\ \text{kg}$$
$$235\ \frac{\text{miles}}{\text{hour}} \times \frac{1\ \text{hour}}{3600\ \text{s}} \times \frac{1609\ \text{m}}{\text{mile}} = 105\ \frac{\text{m}}{\text{s}}$$
$$7\ \text{lb} \rightarrow 3.2\ \text{kg}$$
$$172\ \frac{\text{mile}}{\text{hour}} = 77\ \frac{\text{m}}{\text{s}}$$
$$m_3 = 14.4\ \text{kg} - (4.5\ \text{kg} + 3.2\ \text{kg}) = 6.7\ \text{kg}.$$

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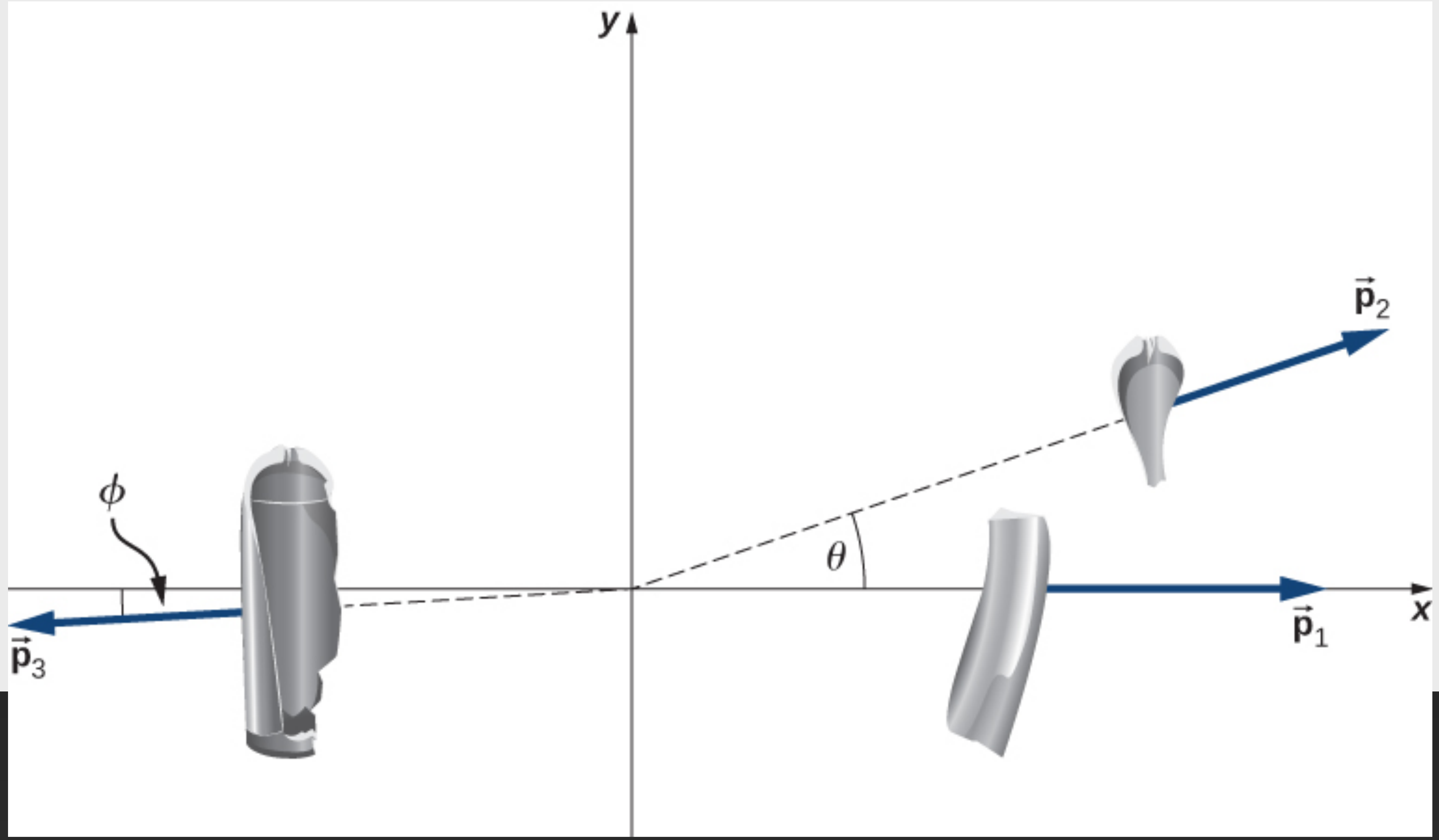

x-direction:

$$\begin{aligned} p_{f,x} &= p_{0,x} \\ p_{1,x} + p_{2,x} + p_{3,x} &= 0 \\ m_1 v_{1,x} + m_2 v_{2,x} + p_{3,x} &= 0 \\ p_{3,x} &= -m_1 v_{1,x} - m_2 v_{2,x} \end{aligned}$$

y-direction:

$$\begin{aligned} p_{f,y} &= p_{0,y} \\ p_{1,y} + p_{2,y} + p_{3,y} &= 0 \\ m_1 v_{1,y} + m_2 v_{2,y} + p_{3,y} &= 0 \\ p_{3,y} &= -m_1 v_{1,y} - m_2 v_{2,y} \end{aligned}$$

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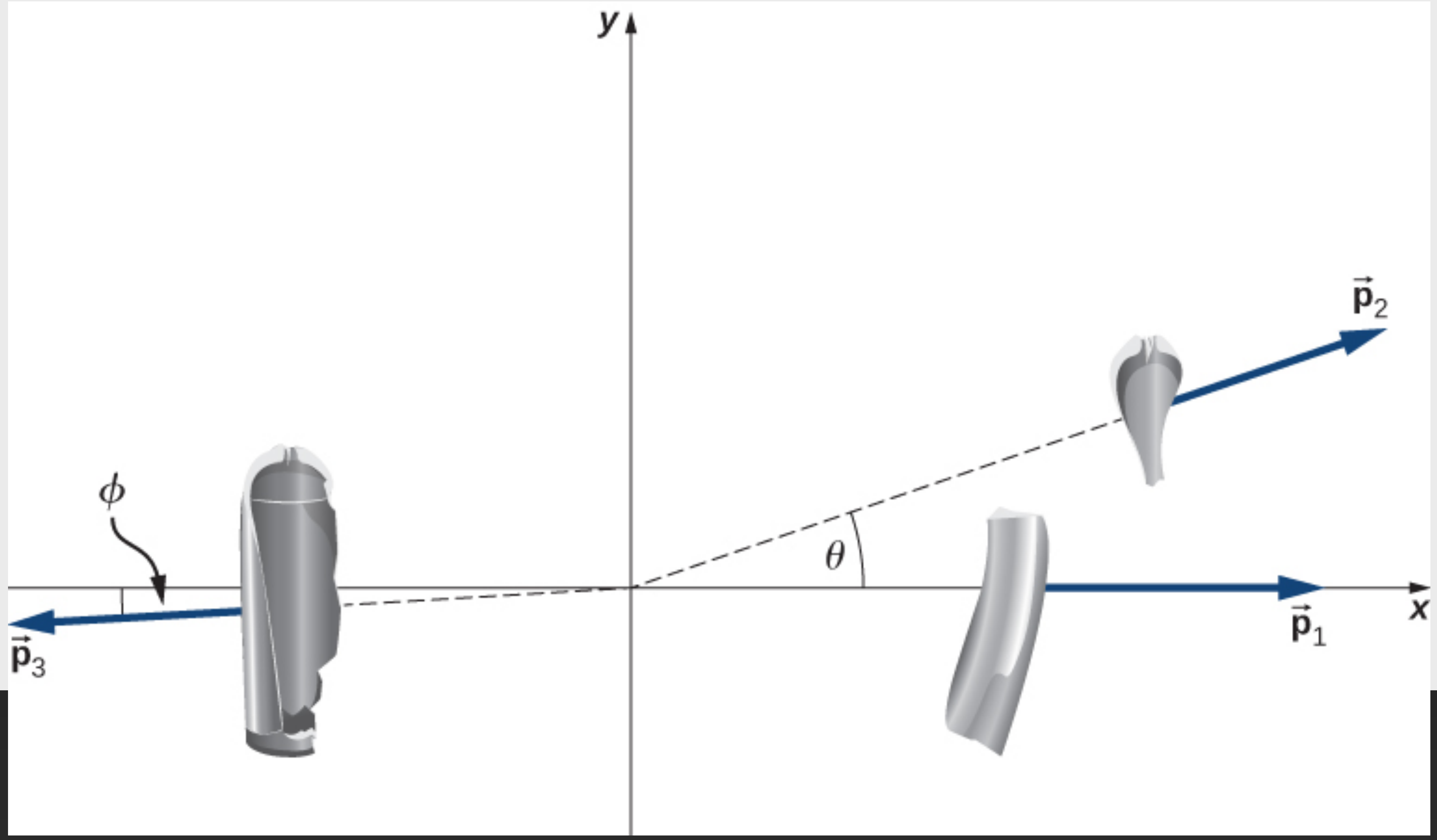

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From our chosen coordinate system, we write the *x*-components as

$$\begin{aligned} p_{3,x} &= -m_1 v_1 - m_2 v_2 \cos\theta \\ &= -(4.5\text{ kg})\left(105\ \tfrac{\text{m}}{\text{s}}\right) - (3.2\text{ kg})\left(77\ \tfrac{\text{m}}{\text{s}}\right)\cos(19°) \\ &= -705\,\tfrac{\text{kg}\cdot\text{m}}{\text{s}}. \end{aligned}$$

For the *y*-direction, we have

$$\begin{aligned} p_{3y} &= 0 - m_2 v_2 \sin\theta \\ &= -(3.2\text{ kg})\left(77\ \tfrac{\text{m}}{\text{s}}\right)\sin(19°) \\ &= -80.2\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}. \end{aligned}$$

This gives the magnitude of p_3:

$$\begin{aligned} p_3 &= \sqrt{p_{3,x}^2 + p_{3,y}^2} \\ &= \sqrt{\left(-705\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}\right)^2 + \left(-80.2\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}\right)} \\ &= 710\ \tfrac{\text{kg}\cdot\text{m}}{\text{s}}. \end{aligned}$$

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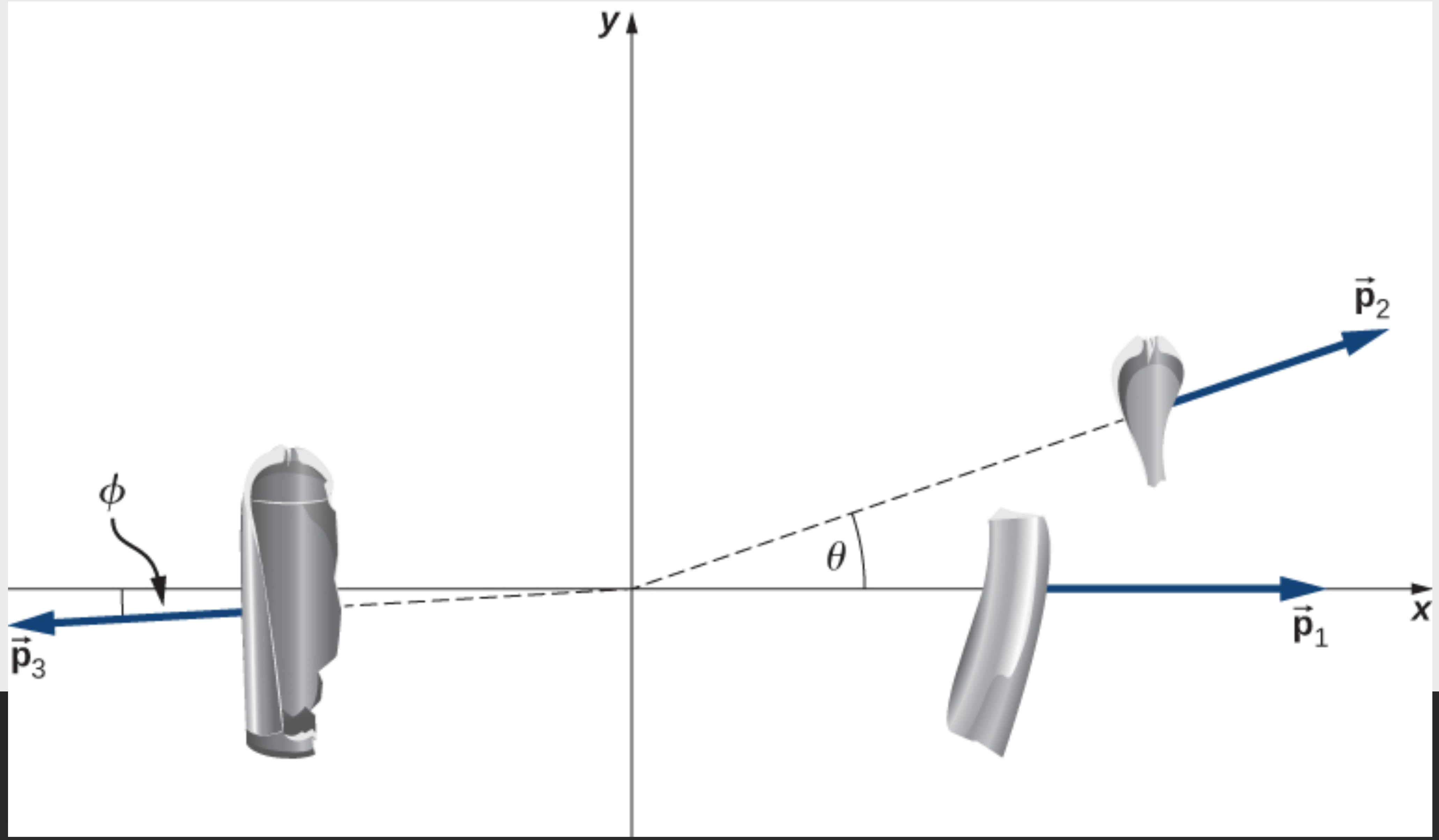

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The velocity of the third piece is therefore

$$v_3 = \frac{p_3}{m_3} = \frac{710\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}{6.7\text{ kg}} = 106\ \frac{\text{m}}{\text{s}}.$$

The direction of its velocity vector is the same as the direction of its momentum vector:

$$\phi = \tan^{-1}\left(\frac{p_{3,y}}{p_{3,x}}\right) = \tan^{-1}\left(\frac{80.2\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}{705\ \frac{\text{kg}\cdot\text{m}}{\text{s}}}\right) = 6.49°.$$

Rocket Propulsion Derivation

Physical Analysis

Here's a description of what happens, so that you get a feel for the physics involved.

- As the rocket engines operate, they are continuously ejecting burned fuel gases, which have both mass and velocity, and therefore some momentum. By conservation of momentum, the rocket's momentum changes by this same amount (with the opposite sign). We will assume the burned fuel is being ejected at a constant rate, which means the rate of change of the rocket's momentum is also constant. By Equation 9.9, this represents a constant force on the rocket.
- However, as time goes on, the mass of the rocket (which includes the mass of the remaining fuel) continuously decreases. Thus, even though the force on the rocket is constant, the resulting acceleration is not; it is continuously increasing.
- So, the total change of the rocket's velocity will depend on the amount of mass of fuel that is burned, and that dependence is not linear.

The problem has the mass and velocity of the rocket changing; also, the total mass of ejected gases is changing. If we define our system to be the rocket + fuel, then this is a closed system (since the rocket is in deep space, there are no external forces acting on this system); as a result, momentum is conserved for this system. Thus, we can apply conservation of momentum to answer the question (Figure 9.33).

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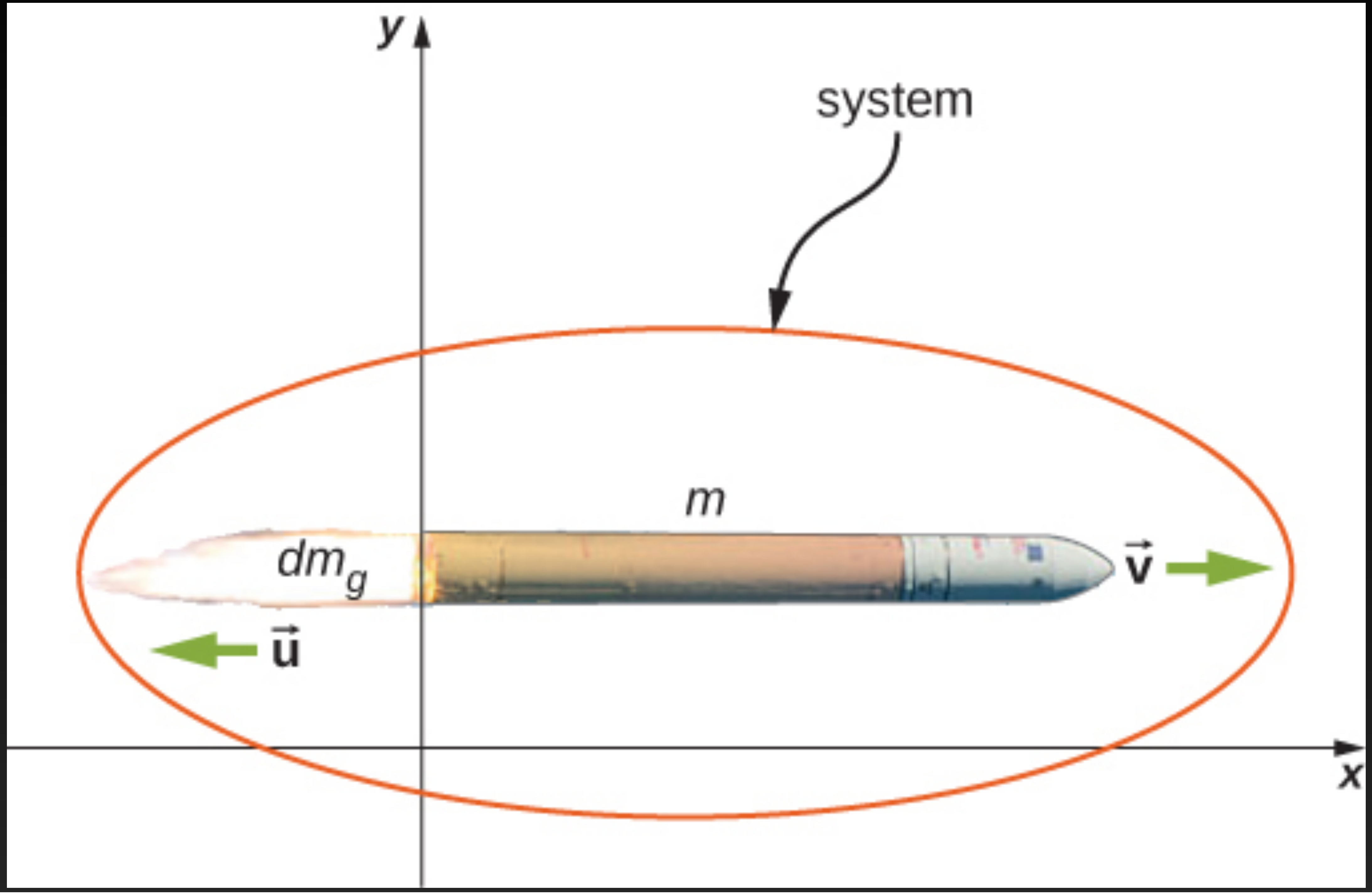

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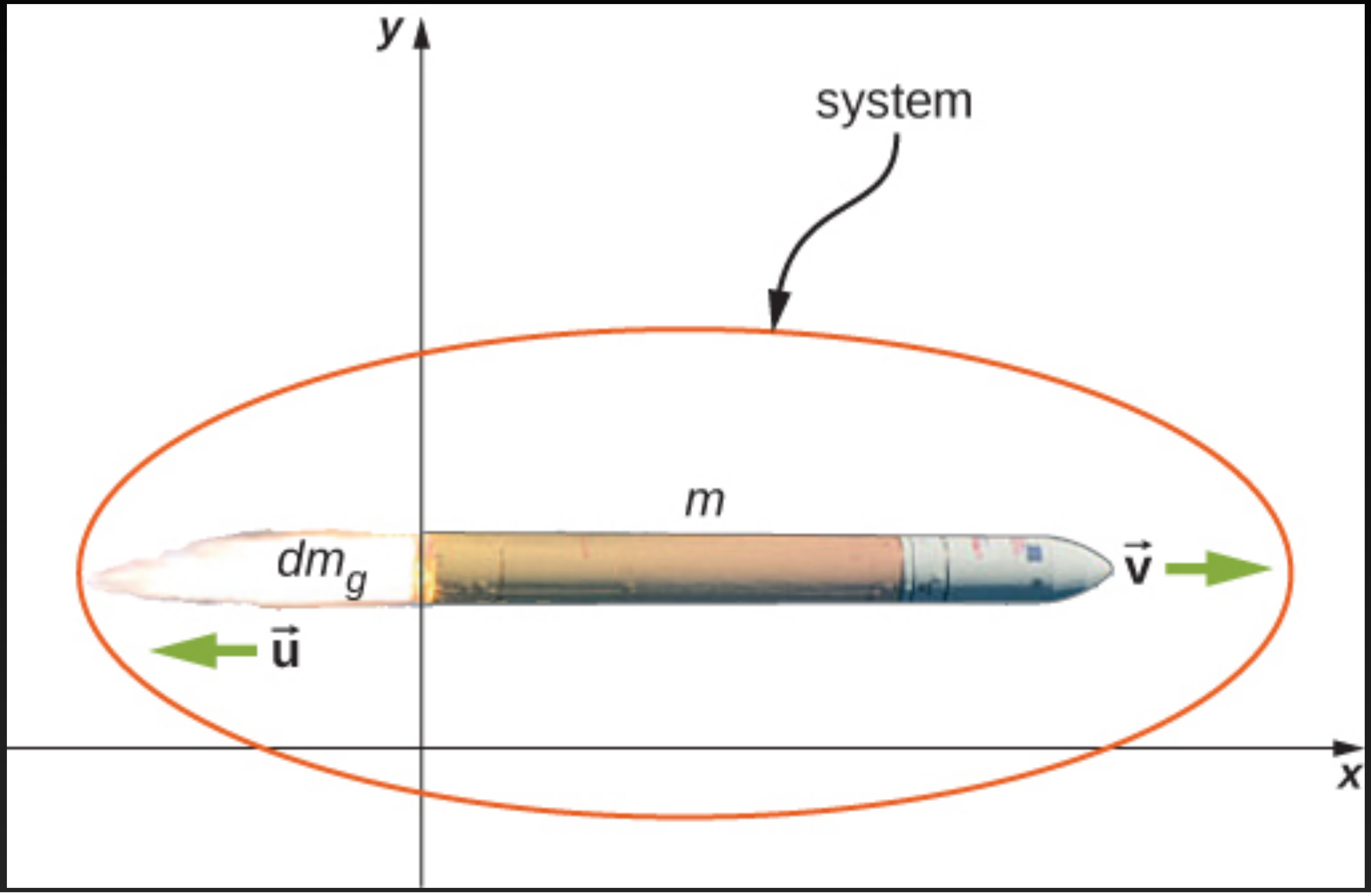

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Since all vectors are in the *x*-direction, we drop the vector notation. Applying conservation of momentum, we obtain

$$
\begin{aligned}
p_i &= p_f \\
mv &= \left(m - dm_g\right)(v + dv) + dm_g(v - u) \\
mv &= mv + mdv - dm_g v - dm_g dv + dm_g v - dm_g u \\
mdv &= dm_g dv + dm_g u.
\end{aligned}
$$

Now, dm_g and dv are each very small; thus, their product $dm_g dv$ is very, very small, much smaller than the other two terms in this expression. We neglect this term, therefore, and obtain:

$$mdv = dm_g u.$$

Our next step is to remember that, since dm_g represents an increase in the mass of ejected gases, it must also represent a decrease of mass of the rocket:

$$dm_g = -dm.$$

Replacing this, we have

$$mdv = -dmu$$

or

$$dv = -u\frac{dm}{m}.$$

Integrating from the initial mass m_0 to the final mass m of the rocket gives us the result we are after:

$$
\begin{aligned}
\int_{v_i}^{v} dv &= -u \int_{m_0}^{m} \frac{1}{m} dm \\
v - v_i &= u \ln\left(\frac{m_0}{m}\right)
\end{aligned}
$$

and thus our final answer is

$$\Delta v = u \ln\left(\frac{m_0}{m}\right). \quad (9.38)$$

Key Equations

Definition of momentum	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
Impulse	$\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}(t)dt$ or $\vec{\mathbf{J}} = \vec{\mathbf{F}}_{ave}\Delta t$
Impulse-momentum theorem	$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$
Average force from momentum	$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$
Instantaneous force from momentum (Newton's second law)	$\vec{\mathbf{F}}(t) = \frac{d\vec{\mathbf{p}}}{dt}$
Conservation of momentum	$\frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} = 0$ or $\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{constant}$
Generalized conservation of momentum	$\sum_{j=1}^{N} \vec{\mathbf{p}}_j = \text{constant}$
Conservation of momentum in two dimensions	$p_{f,x} = p_{1,i,x} + p_{2,i,x}$ $p_{f,y} = p_{1,i,y} + p_{2,i,y}$
Rocket equation	$\Delta v = u \ln\left(\frac{m_i}{m}\right)$

Activity: Worked Problems

35 . Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 1.50×10^5 kg and a velocity of $(0.30 \text{ m/s})\hat{\mathbf{i}}$, and the second having a mass of 1.10×10^5 kg and a velocity of $-(0.12 \text{ m/s})\hat{\mathbf{i}}$. What is their final velocity?

WP 11.1

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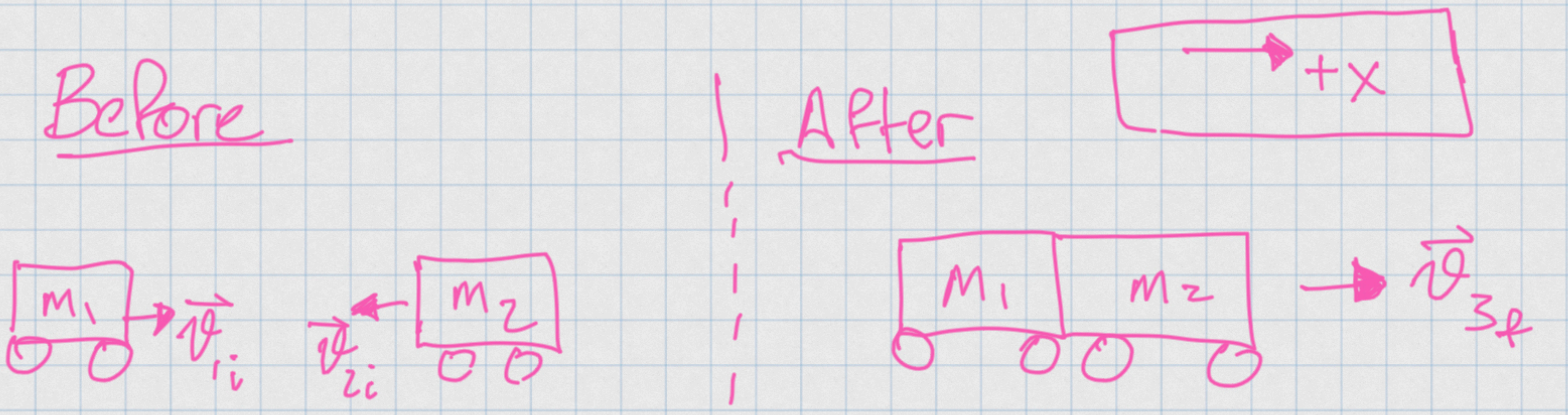
$(0.122 \text{ m/s})\hat{\mathbf{i}}$



By conservation of momentum,

$$P_i = P_f$$

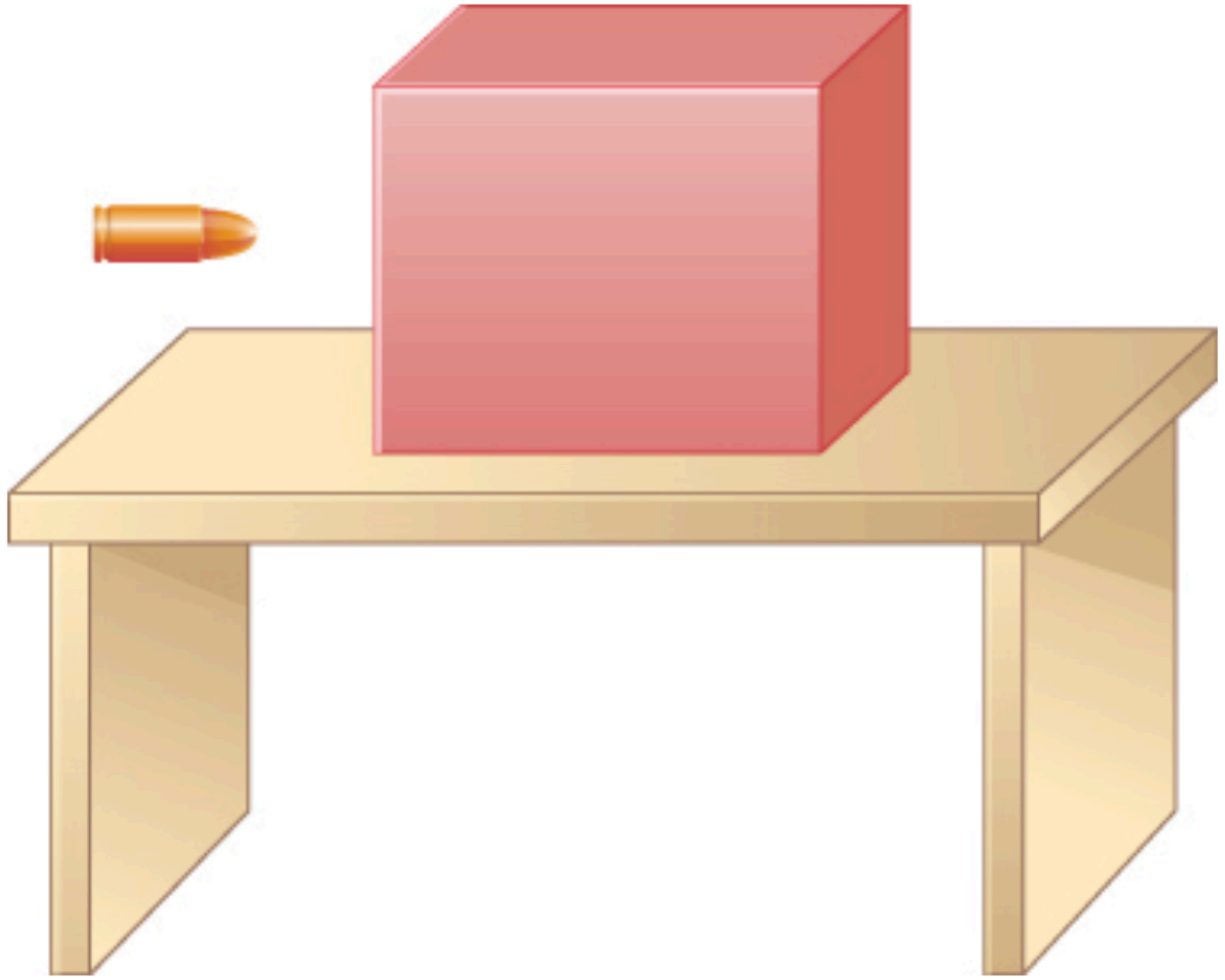
$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2)\vec{v}_{3f}$$

$$\vec{v}_{3f} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

$$\vec{v}_{3f} = 0.122 \text{ m/s } \hat{i}$$

37 . The figure below shows a bullet of mass 200 g traveling horizontally towards the east with speed 400 m/s, which strikes a block of mass 1.5 kg that is initially at rest on a frictionless table.

WP 11.2



After striking the block, the bullet is embedded in the block and the block and the bullet move together as one unit.

a. What is the magnitude and direction of the velocity of the block/bullet combination immediately after the impact?
b. What is the magnitude and direction of the impulse by the block on the bullet?
c. What is the magnitude and direction of the impulse from the bullet on the block?
d. If it took 3 ms for the bullet to change the speed from 400 m/s to the final speed after impact, what is the average force between the block and the bullet during this time?

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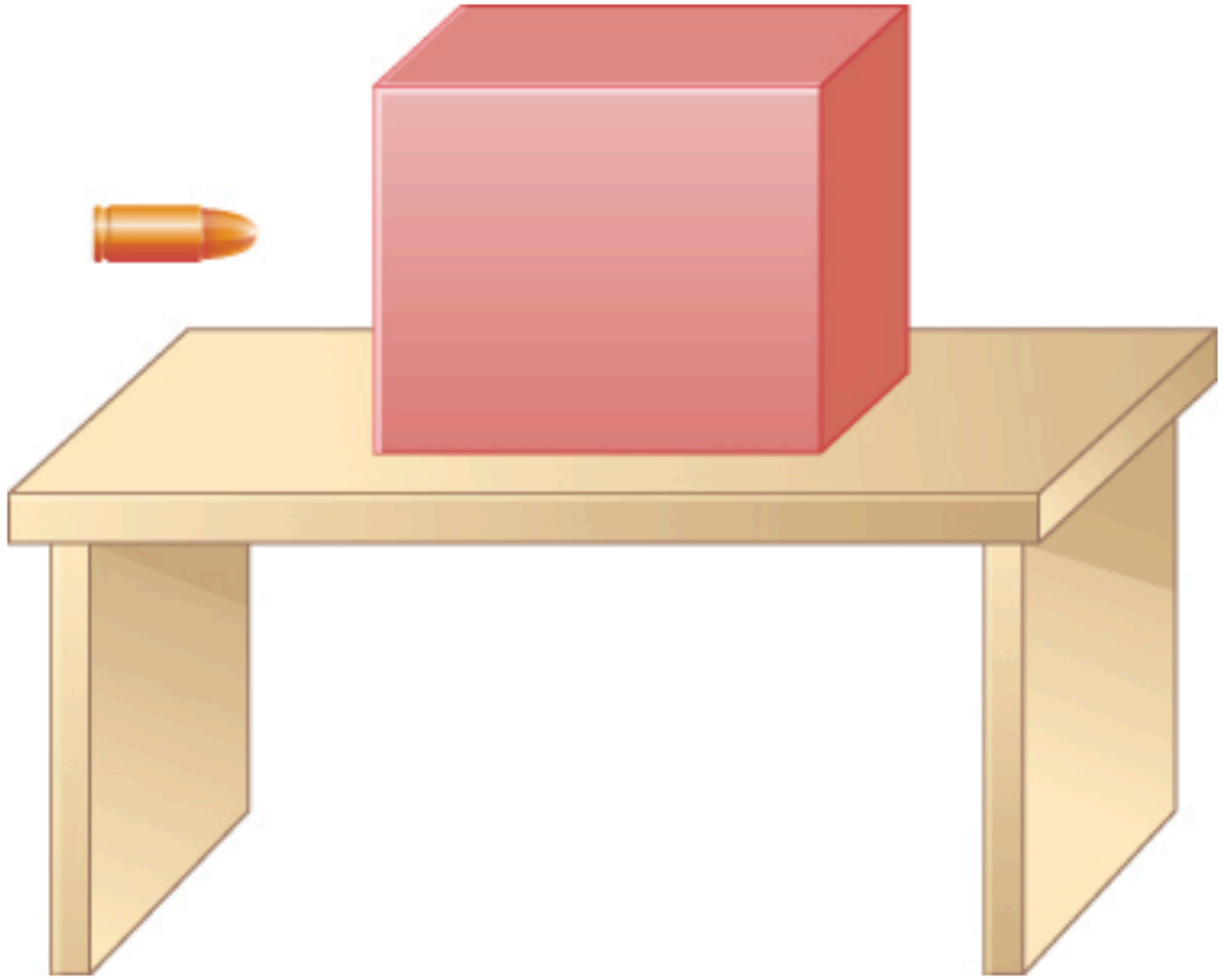
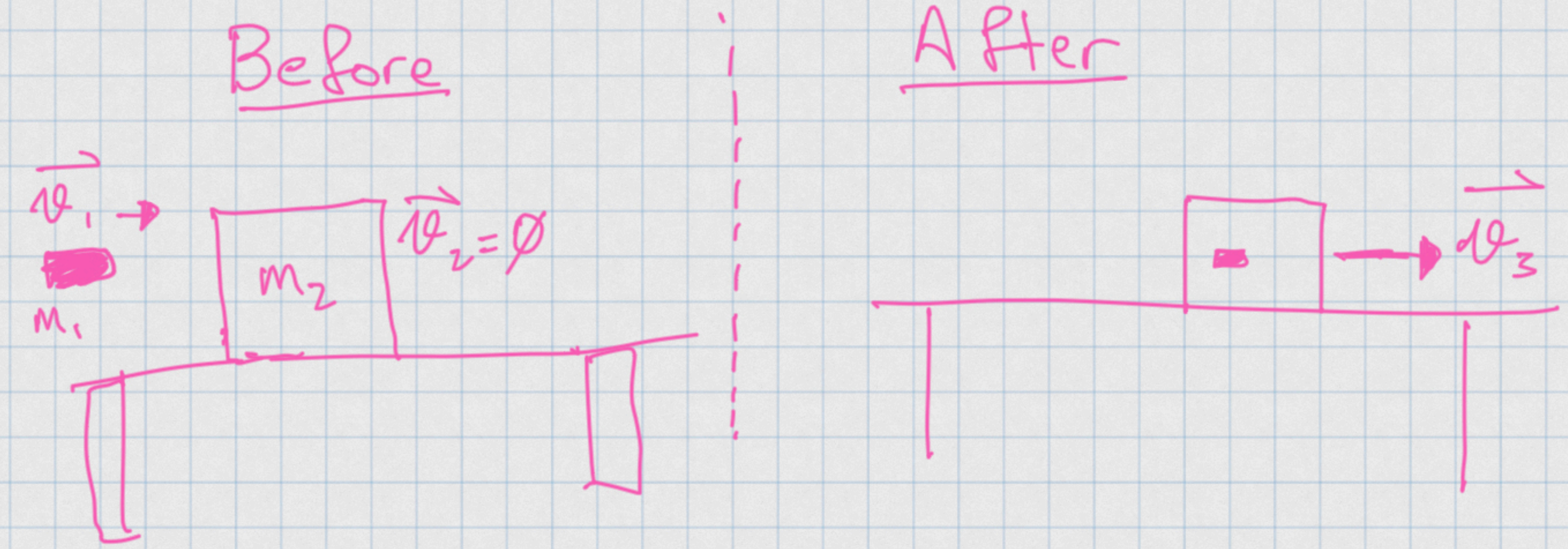




a) From conservation of momentum,

$$\vec{P}_i = \vec{P}_f$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{v}_3 \quad (\vec{v}_2 \rightarrow \emptyset)$$

$$\boxed{\vec{v}_3 = \left(\frac{m_1}{m_1+m_2}\right)\cdot\vec{v}_1}$$

47 m/s $\hat{i}$

b) Impulse is just the change in momentum:

$$\vec{J} = m\Delta\vec{v} \qquad \Delta v = v_f - v_i$$

$$= (1.5\,kg)\cdot(47\,m/s)$$

$$\boxed{\vec{J} = 70.6\ kg\ m/s\ (N\cdot s)}$$

c) $\vec{J} = m\Delta\vec{v}$ $\qquad \Delta v = v_f - v_i = (400 - 47)\,m/s$

$$= 0.2\,kg\,(-353\,m/s)$$

$$\boxed{\vec{J} = -70.6\ kg\cdot m/s\ (N\cdot s)}$$

Impulse from block on bullet is negative bullet on block.

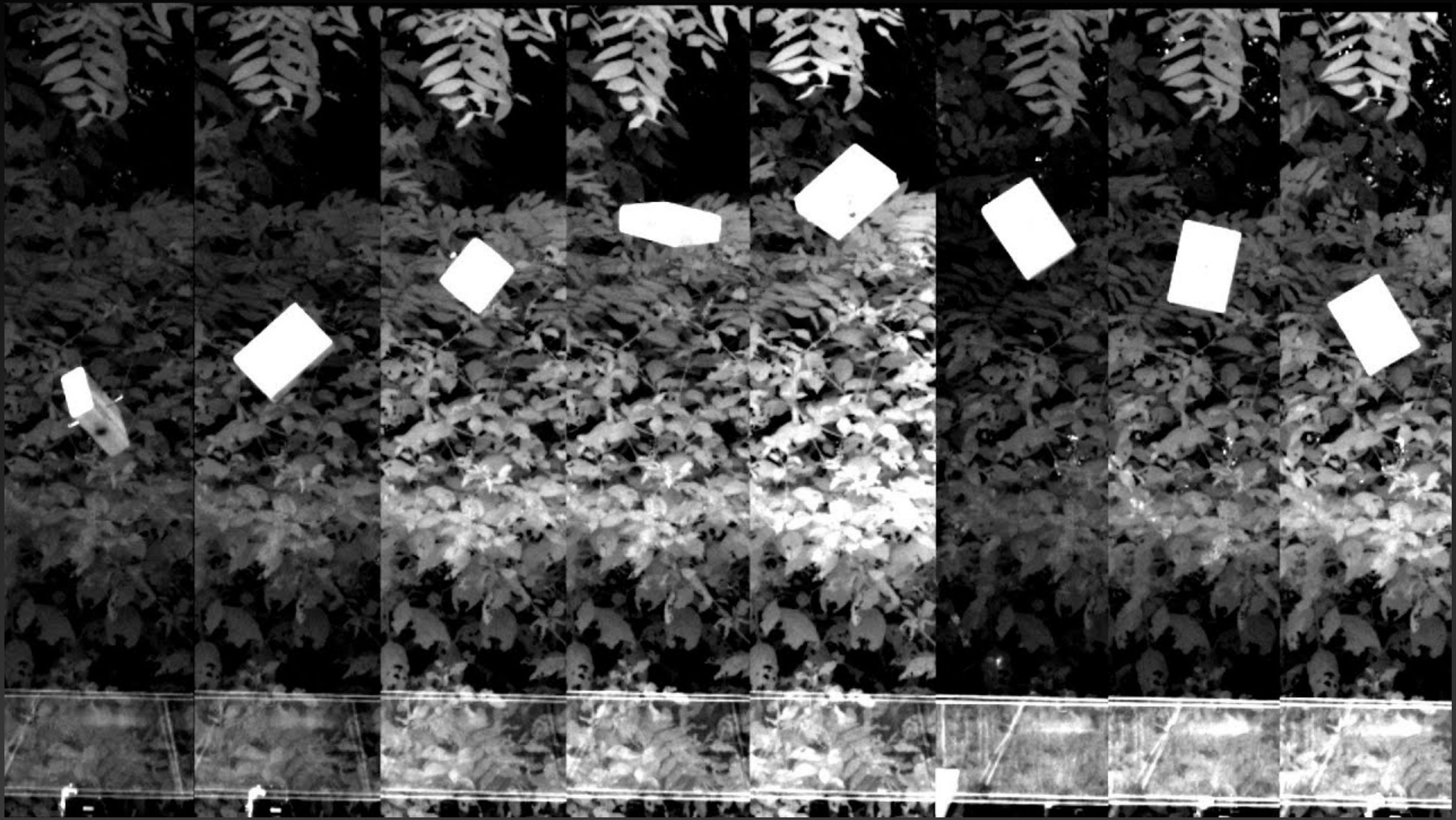
d) Given the period of time over which $\vec{J}$ is imparted:

$$\vec{J} = m\Delta v \quad \text{and} \quad J = \vec{F}_{Ave}\Delta t$$

Revisiting the Bullet Block problem…

Preview of what's left

Preview of what's left



Did we just violate Conservation of Energy?

Did we just violate Conservation of Energy?

No! Physics is safe…

Energy is Conserved
Linear Momentum and Angular Momentum is separately conserved.

Key point: Bullet doesn't go as far into the block!

See you next class!

Attribution

This resource was significantly adapted from the Open Stax Instructor Slides provided by Rice University. It is released under a CC-BY 4.0 license.