You can draw here

# Physics 111 - Class 11B
# Momentum & Impulse

Do not draw in/on this box!

November 17, 2021

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# Class Outline

- Logistics / Announcements
- Ball Race
- Chapter 9 Section Summary
- Clicker Questions
- Worked Problems

# Logistics/Announcements

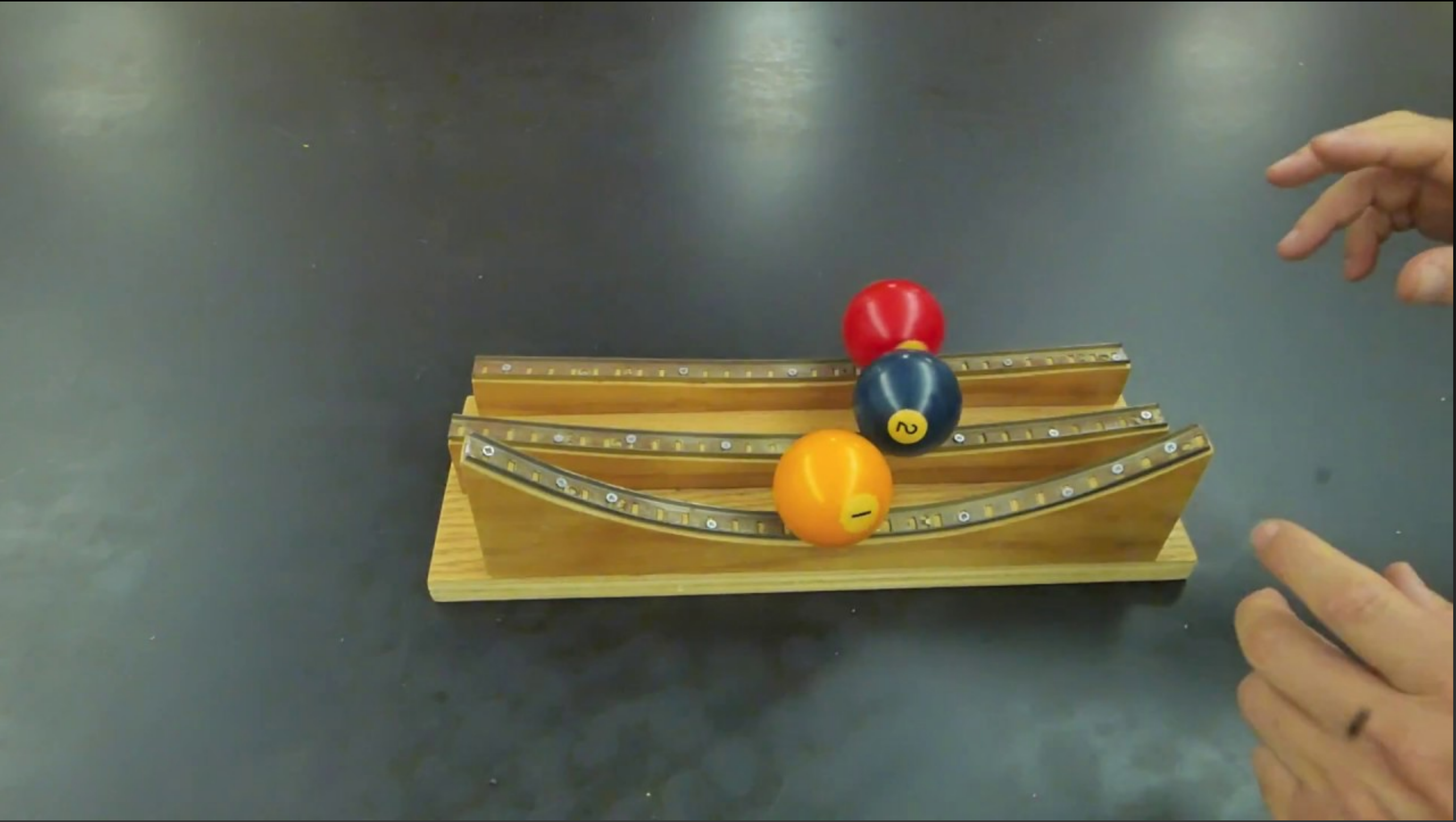
- Lab this week: Lab 7
- HW9 due this week on Thursday at 6 PM
- Learning Log 9 due on Saturday at 6 PM
- HW and LL deadlines have a 48 hour grace period
- Test/Bonus Test: Bonus Test 4 available this week (Chapters 7 & 8)
    - Test Window: Friday 6 PM - Sunday 6 PM

# Which Ball reaches the end first?

## Ball Race

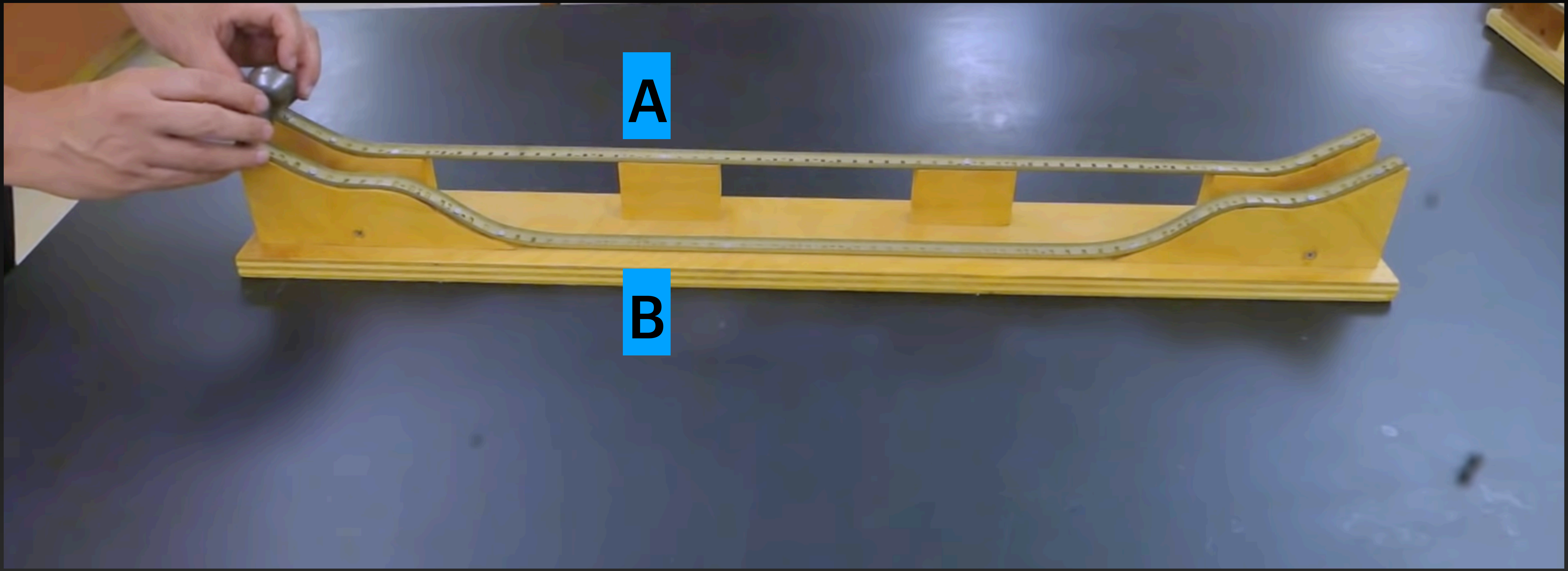
Two identical balls, Ball A and Ball B are launched with the same initial velocity $v$ along a pair of tracks. The first track with Ball A, is a straight track. The second track with Ball B, has a "U"-shaped dip in the middle so the ball goes down and then back up.


Which ball reaches the end of the track first, if friction is neglected?



Video Reference: Physics marble track review part one

# Which Ball reaches the end first?

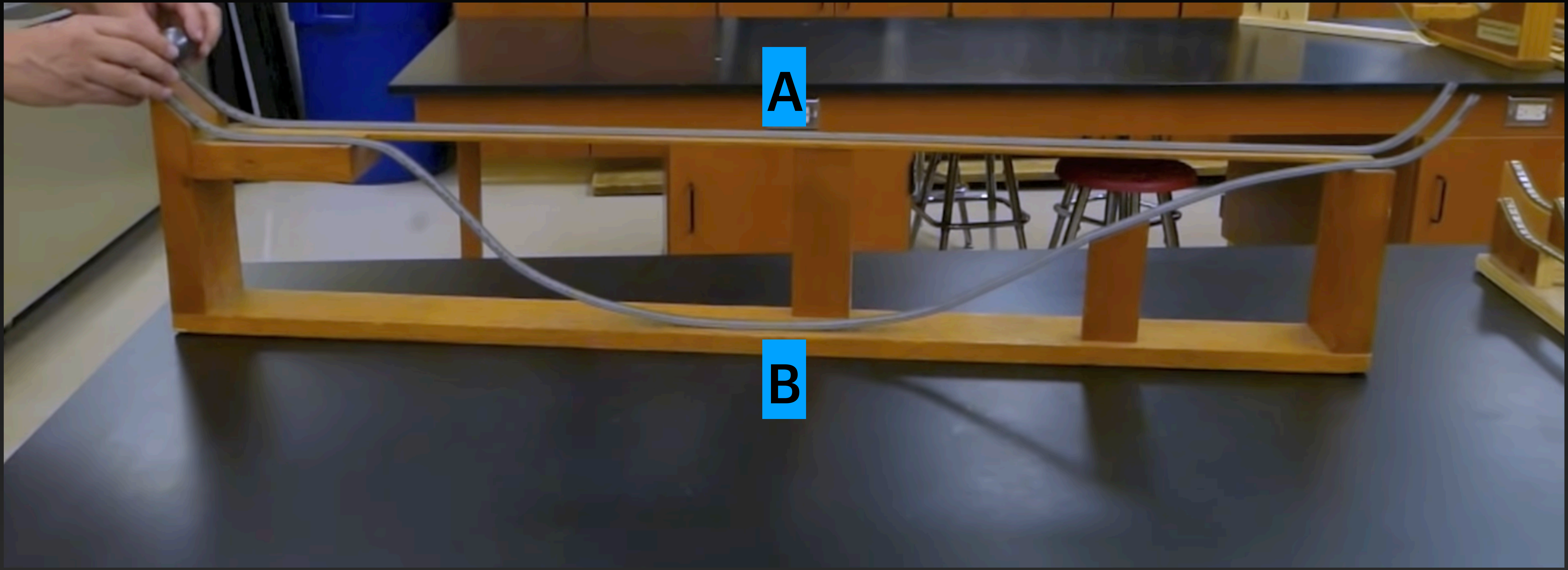



A B C D

C - Reach the end at the same time

D - I don't know!

# Which Ball reaches the end first?

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# Which Ball reaches the end first?


A B C D

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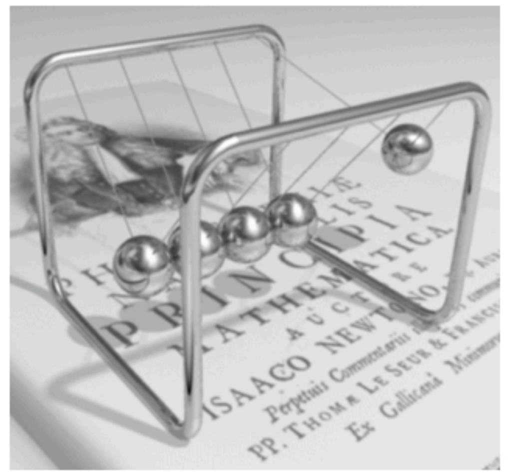


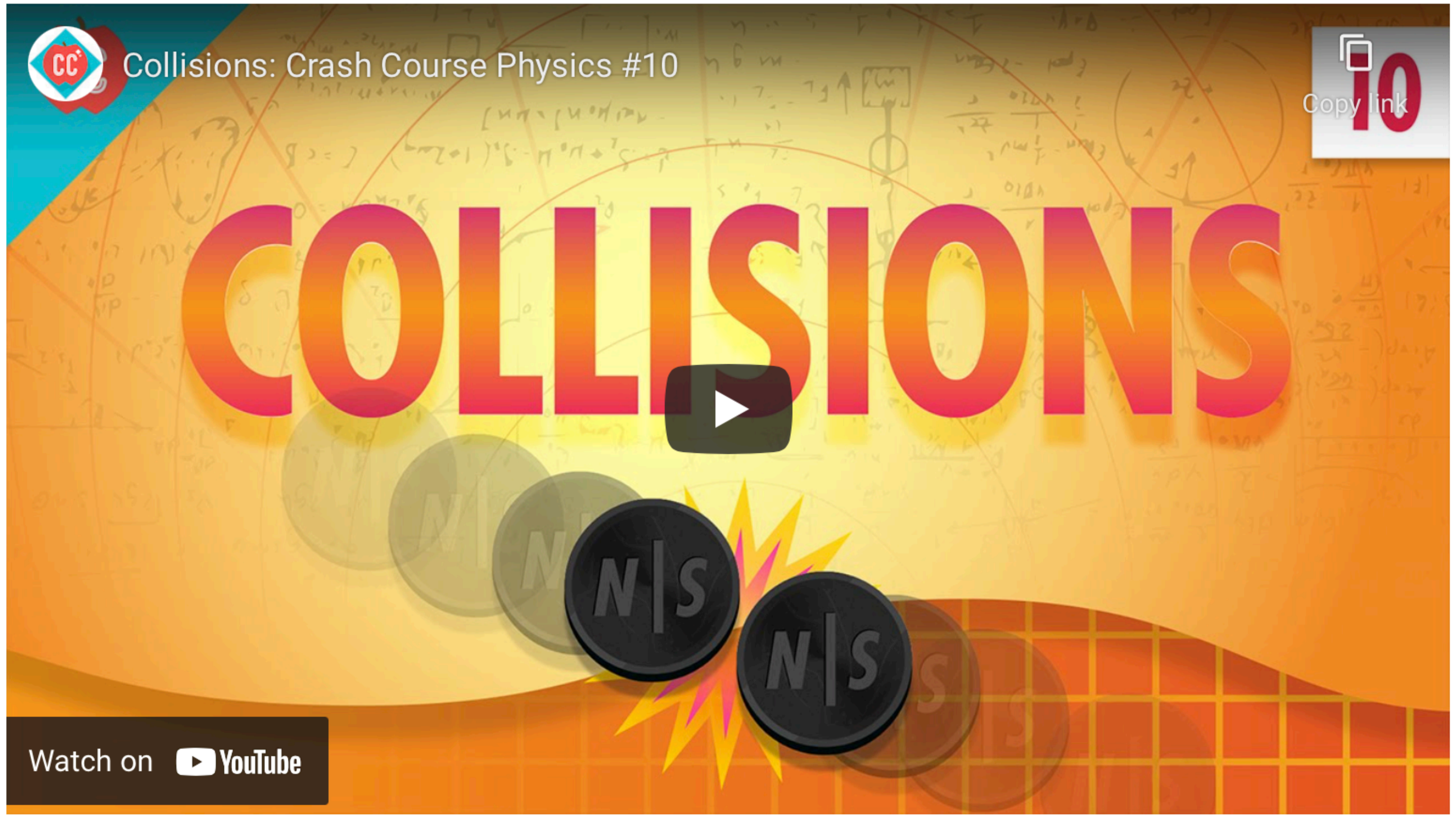
Physics 111





# Content Summary from Crash Course Physics

**Collisions**






# Required Videos

**1. You Can't Run From Momentum! (a momentum introduction)**

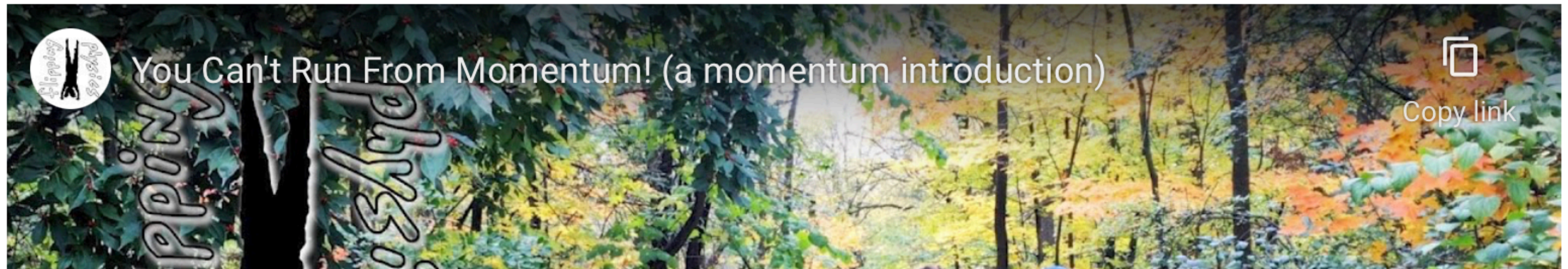



**Figure 9.1** The concepts of impulse, momentum, and center of mass are crucial for a major-league baseball player to successfully get a hit. If he misjudges these quantities, he might break his bat instead. (credit: modification of work by "Cathy T"/Flickr)

## Chapter Outline



The concepts of work, energy, and the work-energy theorem are valuable for two primary reasons: First, they are powerful computational tools, making it much easier to analyze complex physical systems than is possible using

# Monday’s Class (cont’d)

9.2 Impulse and Collisions
9.4 Types of Collisions
9.5 Collisions in multiple dimensions

# Impulse

The product of a force and a time interval (over which that force acts) is called **impulse**, and is given the symbol $\vec{\mathbf{J}}$.

## IMPULSE

Let $\vec{\mathbf{F}}(t)$ be the force applied to an object over some differential time interval *dt* (Figure 9.6). The resulting impulse on the object is defined as

$$d\vec{\mathbf{J}} \equiv \vec{\mathbf{F}}(t)dt. \tag{9.2}$$

The total impulse over the interval $t_f - t_i$ is

$$\vec{\mathbf{J}} \equiv \int_{t_i}^{t_f} \vec{\mathbf{F}}(t)dt. \tag{9.3}$$

## IMPULSE-MOMENTUM THEOREM

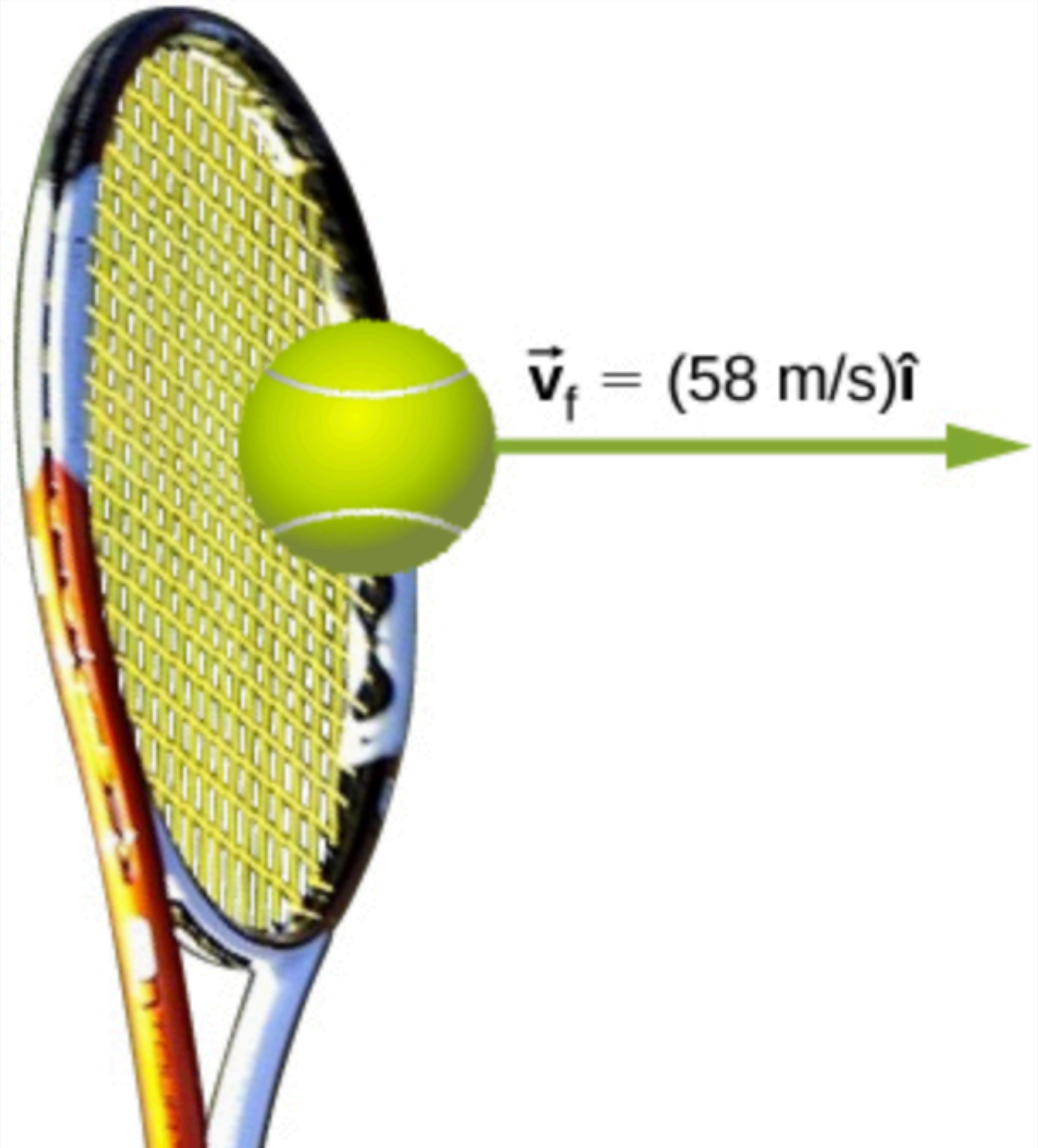
An impulse applied to a system changes the system's momentum, and that change of momentum is exactly equal to the impulse that was applied:

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}. \tag{9.7}$$

EXAMPLE 9.5

### Calculating Force: Venus Williams' Tennis Serve

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet? Assume that the ball's speed just after impact is 58 m/s, as shown in Figure 9.13, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms.



Example 9.5

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### Solution

To determine the change in momentum, insert the values for the initial and final velocities into the equation above:

$$\begin{aligned}\Delta p &= m\left(v_{\mathrm{f}} - v_{\mathrm{i}}\right)\\ &= (0.057\ \mathrm{kg})\,(58\ \mathrm{m/s} - 0\ \mathrm{m/s})\\ &= 3.3\ \frac{\mathrm{kg{\cdot}m}}{\mathrm{s}}.\end{aligned}$$

Now the magnitude of the net external force can be determined by using

$$F = \frac{\Delta p}{\Delta t} = \frac{3.3\ \frac{\mathrm{kg{\cdot}m}}{\mathrm{s}}}{5.0 \times 10^{-3}\ \mathrm{s}} = 6.6 \times 10^{2}\ \mathrm{N}.$$

where we have retained only two significant figures in the final step.

# Clicker Questions

# Key Equations

| | |
|---|---|
| External forces | $\vec{\mathbf{F}}_{\text{ext}} = \sum_{j=1}^{N} \frac{d\vec{\mathbf{p}}_j}{dt}$ |
| Newton's second law for an extended object | $\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}_{\text{CM}}}{dt}$ |
| Acceleration of the center of mass | $\vec{\mathbf{a}}_{\text{CM}} = \frac{d^2}{dt^2}\left(\frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{r}}_j\right) = \frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{a}}_j$ |
| Position of the center of mass for a system of particles | $\vec{\mathbf{r}}_{\text{CM}} \equiv \frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{r}}_j$ |
| Velocity of the center of mass | $\vec{\mathbf{v}}_{\text{CM}} = \frac{d}{dt}\left(\frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{r}}_j\right) = \frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{v}}_j$ |
| Position of the center of mass of a continuous object | $\vec{\mathbf{r}}_{\text{CM}} \equiv \frac{1}{M}\int \vec{\mathbf{r}}\, dm$ |
| Rocket equation | $\Delta v = u \ln\left(\frac{m_{\text{i}}}{m}\right)$ |

# CQ.11.1

What is the momentum of a bowling ball with mass $5\,\mathrm{kg}$ and velocity $10\,\mathrm{m/s}$?

a) $0.5\,\mathrm{kg}\cdot\mathrm{m/s}$

b) $2\,\mathrm{kg}\cdot\mathrm{m/s}$

c) $15\,\mathrm{kg}\cdot\mathrm{m/s}$

d) $50\,\mathrm{kg}\cdot\mathrm{m/s}$

A B C D E

# CQ.11.1

**What is the momentum of a bowling ball with mass $5\,\text{kg}$ and velocity $10\,\text{m/s}$?**

a) $0.5\,\text{kg}\cdot\text{m/s}$

b) $2\,\text{kg}\cdot\text{m/s}$

c) $15\,\text{kg}\cdot\text{m/s}$

✓ d) $50\,\text{kg}\cdot\text{m/s}$

**Detailed solution:** $p = mv = 50\,\text{kg}\cdot\text{m/s}$

A B C D E

# CQ.11.2

**When the momentum of an object increases with respect to time, what is true of the net force acting on it?**

a) It is zero, because the net force is equal to the rate of change of the momentum.

b) It is zero, because the net force is equal to the product of the momentum and the time interval.

c) It is nonzero, because the net force is equal to the rate of change of the momentum.

d) It is nonzero, because the net force is equal to the product of the momentum and the time interval.

A B C D E

# CQ.11.2

**When the momentum of an object increases with respect to time, what is true of the net force acting on it?**

a) It is zero, because the net force is equal to the rate of change of the momentum.

b) It is zero, because the net force is equal to the product of the momentum and the time interval.

✓ c) It is nonzero, because the net force is equal to the rate of change of the momentum.

d) It is nonzero, because the net force is equal to the product of the momentum and the time interval.

**Detailed solution:** If the object's velocity is constant, the momentum would be proportional to the mass of the object because momentum is defined as the product of the mass and the velocity of the moving object.

A B C D E

# CQ.11.3

For how long should a force of $130.0\,\mathrm{N}$ be applied to an object of mass $50.0\,\mathrm{kg}$ to change its speed from $20.0\,\mathrm{m/s}$ to $60.0\,\mathrm{m/s}$?

a) $0.031\ \mathrm{s}$

b) $0.065\ \mathrm{s}$

c) $15.4\ \mathrm{s}$

d) $40.0\ \mathrm{s}$

A B C D E

# CQ.11.3

For how long should a force of $130.0\,\mathrm{N}$ be applied to an object of mass $50.0\,\mathrm{kg}$ to change its speed from $20.0\,\mathrm{m/s}$ to $60.0\,\mathrm{m/s}$?

a) $0.031\ \mathrm{s}$

b) $0.065\ \mathrm{s}$

✓ c) $15.4\ \mathrm{s}$

d) $40.0\ \mathrm{s}$

**Detailed solution:** $\Delta p = m\Delta v = 2.00 \times 10^3\ \mathrm{kg \cdot m/s}$ $\Delta p = F_{\mathrm{net}}\Delta t$ $\Delta t = 15.4\ \mathrm{s}$

A B C D E

# CQ.11.4

**Cars these days have parts that can crumple or collapse in the event of an accident. What is the advantage of this?**

a) It reduces injury to the passengers by increasing the time of impact.

b) It reduces injury to the passengers by decreasing the time of impact.

c) It reduces injury to the passengers by increasing the change in momentum.

d) It reduces injury to the passengers by decreasing the change in momentum.

A B C D E

# CQ.11.4

**Cars these days have parts that can crumple or collapse in the event of an accident. What is the advantage of this?**

✓ a) It reduces injury to the passengers by increasing the time of impact.

b) It reduces injury to the passengers by decreasing the time of impact.

c) It reduces injury to the passengers by increasing the change in momentum.

d) It reduces injury to the passengers by decreasing the change in momentum.

**Detailed solution:** It increases the duration over which the force of impact acts on the car, thus reducing injury to the passengers.

A B C D E

# CQ.11.5

A person with mass $65\ \mathrm{kg}$, standing still, throws an object at $4\ \mathrm{m/s}$. If the recoil velocity of the person is $3.5\ \mathrm{m/s}$, what is the mass of the object? Assume the surface to be frictionless.

a) $-65\ \mathrm{kg}$

b) $-56.8\ \mathrm{kg}$

c) $56.8\ \mathrm{kg}$

d) $65\ \mathrm{kg}$

A B C D E

# CQ.11.5

**A person with mass $65\ \mathrm{kg}$, standing still, throws an object at $4\ \mathrm{m/s}$. If the recoil velocity of the person is $3.5\ \mathrm{m/s}$, what is the mass of the object? Assume the surface to be frictionless.**

a) $-65\ \mathrm{kg}$

b) $-56.8\ \mathrm{kg}$

✓ c) $56.8\ \mathrm{kg}$

d) $65\ \mathrm{kg}$

A B C D E

# Wednesday’s Class

## 9.4 Types of Collisions

# Types of Collisions

- There are several possibilities of interaction between objects if momentum is conserved:


  - Let’s brainstorm all the possible options that can occur if two objects “interact” ; let’s avoid labeling them for now…

Before Collision

After Collision

# Elastic Collisions

In **elastic** collisions all of the energy remains as kinetic energy — no energy is lost to other forms. This means that both kinetic energy and momentum are conserved.

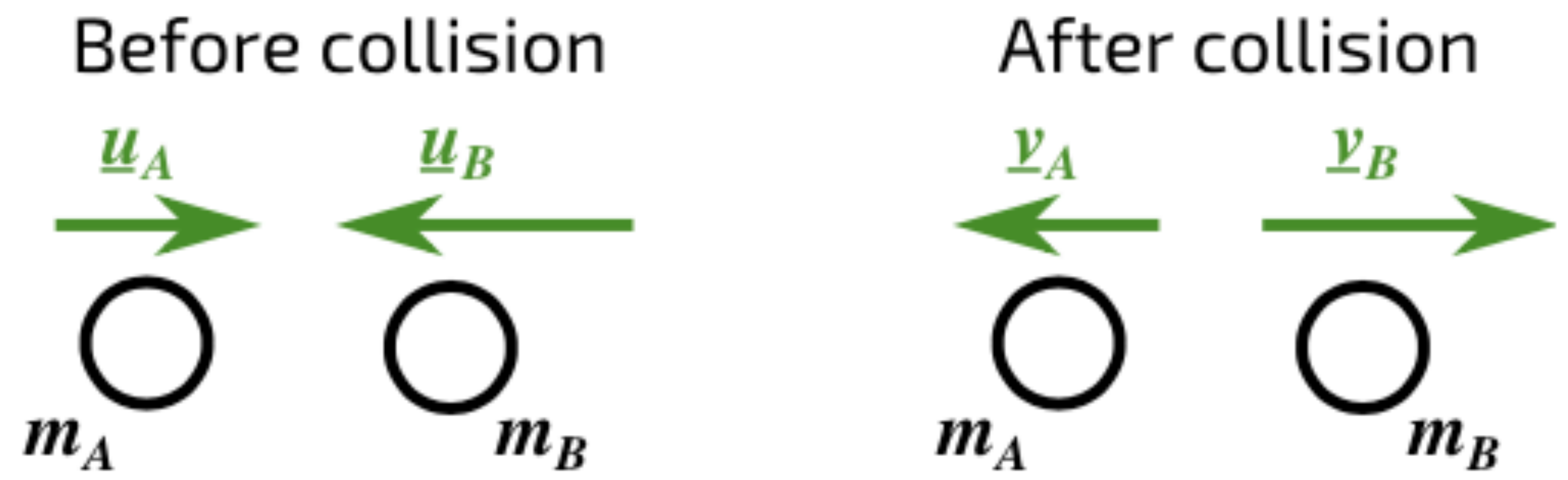



**Figure 2:** An elastic collision between two particles.

**Figure 2** shows a simple case. Before the collision, particle A with mass $m_A$ is moving towards particle B with a speed $u_A$, while particle B with mass $m_B$ is moving towards particle B with a speed $u_B$. The collision is elastic, so both momentum and kinetic energy must be conserved.

# Inelastic Collisions

In inelastic collisions, some kinetic energy is converted to another form. In fully inelastic collisions the maximum possible kinetic energy is lost and the objects stick together. However in many inelastic collisions this is not the case — only some kinetic energy is lost.

In an inelastic collision:

*Kinetic Energy before collision = Kinetic Energy after collision + Energy converted into other forms*

We can use this along with the conservation of momentum, which is always conserved, to work out the motion of objects after the collision.

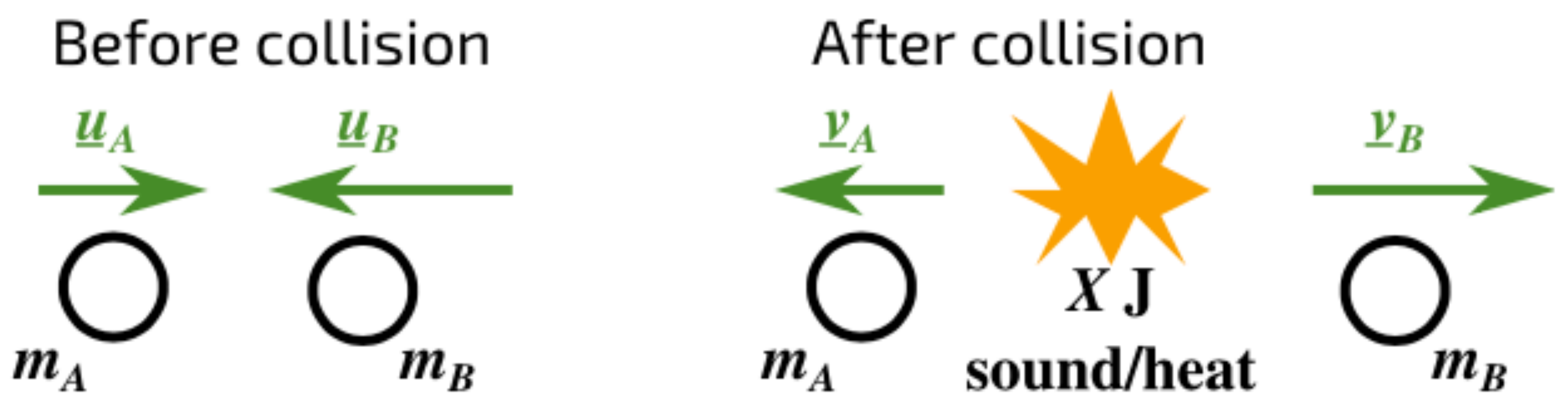



**Figure 3:** An inelastic collision between two particles, releasing X J of sound and heat.

**Figure 3** shows an inelastic collision between two particles, both of mass $m$, in which $\Delta K = X$ J of sound and heat are produced. The particle motion involved in the sound and heat has net zero momentum.

# Completely Inelastic Collisions

The easiest collisions to analyse are **completely inelastic** collisions, where objects stick together after colliding. The two objects have the same final velocity, which we can calculate by conservation of momentum.

Energy is converted into other forms in the collision, so we don't have to worry about conserving kinetic energy.

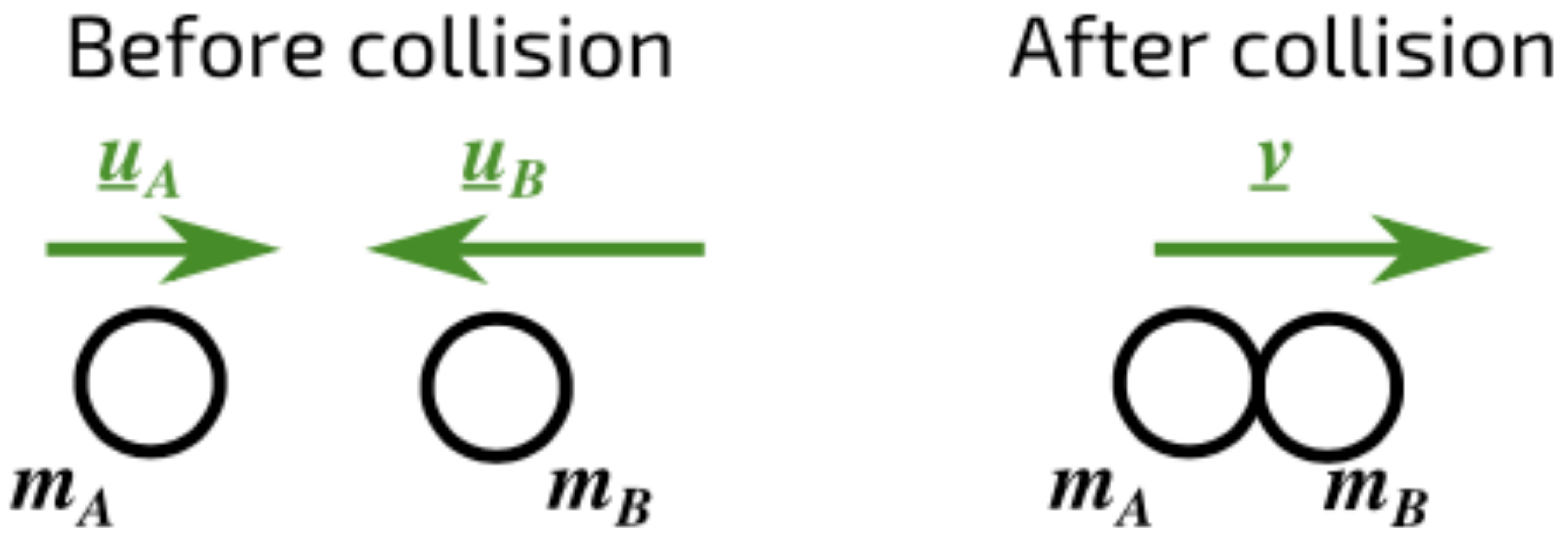



**Figure 1:** A completely inelastic collision between two particles.

**Figure 1** shows a simple case. Before the collision, particle A with mass $m_{\mathrm{A}}$ is moving towards particle B with a speed $u_{\mathrm{A}}$, and particle B with mass $m_{\mathrm{B}}$ is moving towards particle A with a speed $u_{\mathrm{B}}$. The total momentum (taking to the right as positive) is $p = m_{\mathrm{A}} u_{\mathrm{A}} - m_{\mathrm{B}} u_{\mathrm{B}}$ .

# Explosions

It is also possible to *increase* the kinetic energy after a "collision" if another form of energy is converted into kinetic energy. This commonly occurs in explosions, in which chemical energy is converted into kinetic energy. In this case:

*Kinetic Energy before collision + Chemical energy released during explosion = Kinetic Energy after collision.*

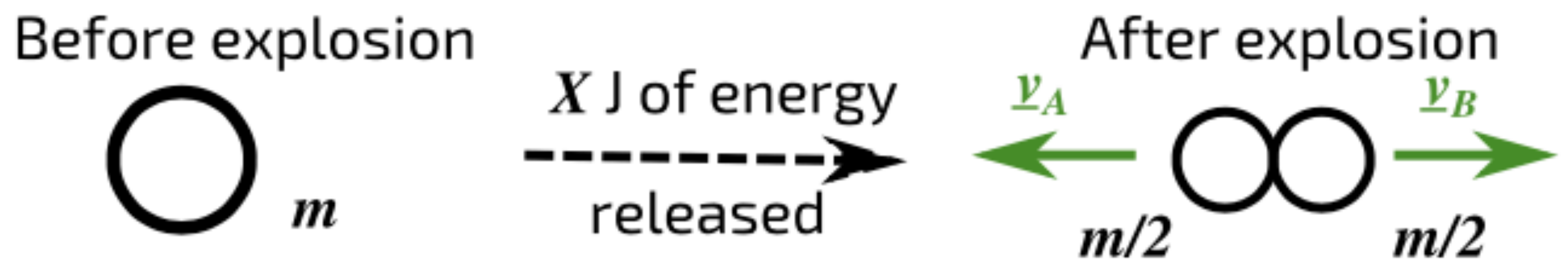



Figure 4: An explosion in which a mass $m$ splits into two equal masses of mass $m/2$.

Figure 4 shows an explosion where a stationary mass $m$ splits into two equal masses of mass $\frac{m}{2}$, with velocities $\underline{\boldsymbol{v}}_{\mathrm{A}}$ and $\underline{\boldsymbol{v}}_{\mathrm{B}}$, releasing $\Delta K = X$ J of kinetic energy.

# Solving Problems in the Zero Momentum Frame

The previous section described an elastic collision between two particles in the zero momentum frame (ZMF). After the collision, they move away from each other with the same speeds as they had before the collision. This can be used to solve collision problems in other frames without having to solve simultaneous equations for conservation of energy and momentum. This is why the ZMF is useful.

The **laboratory frame** is the frame in which the collision happens as viewed by a stationary scientist watching the event. The **ZMF** is a frame moving at a specific velocity - think of it as an observer moving at this speed. More detail on moving between frames of reference can be found at the Frames of Reference concept page.

We start by looking at an elastic head-on 1D collision involving particle A of mass $m_{\mathrm{A}} = m$ travelling with an initial velocity $\underline{u}$, and a stationary particle B of mass $m_{\mathrm{B}} = 2m$, as shown in Figure 7.

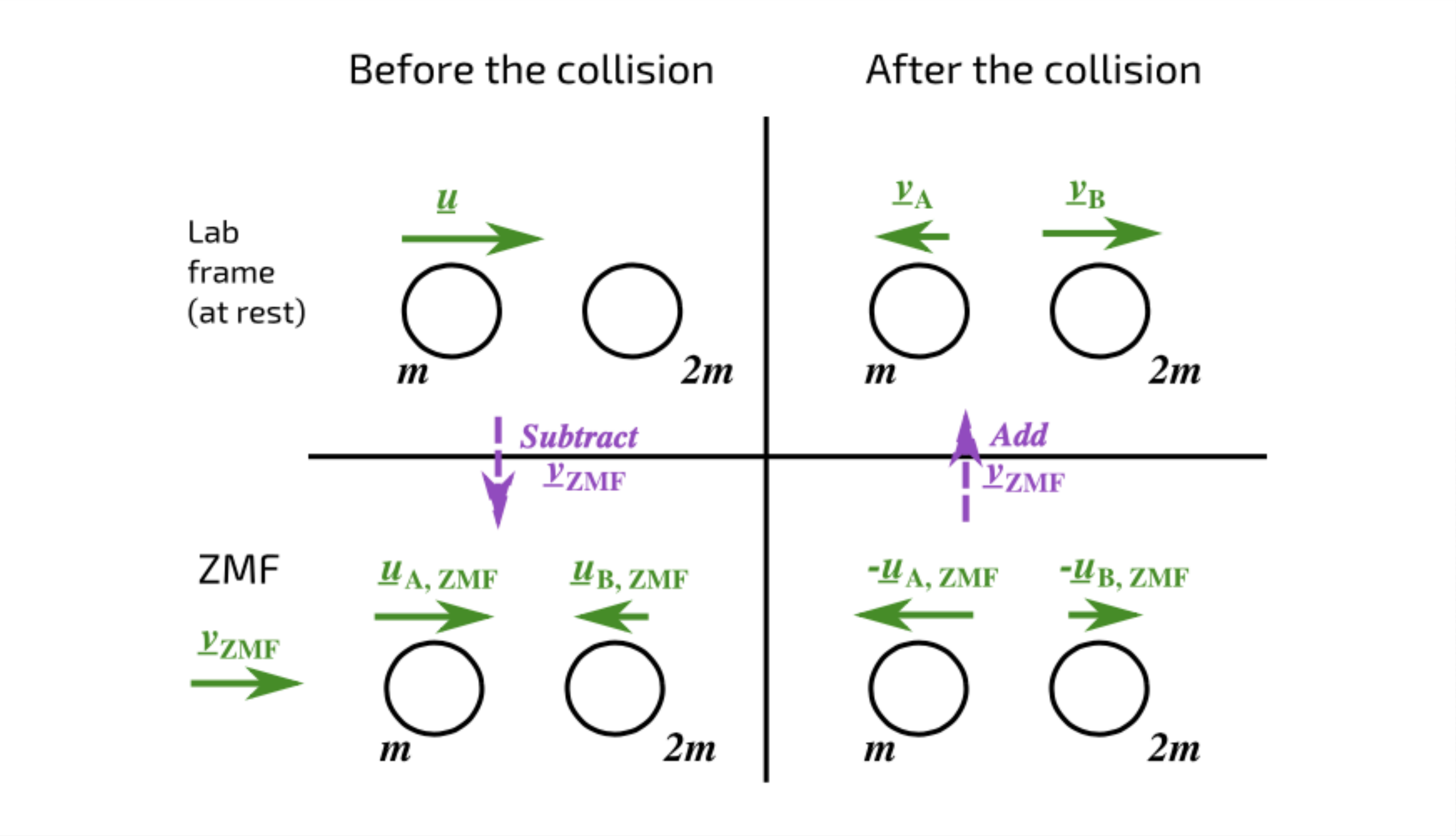



Figure 7: A 1D collision using the Zero Momentum Frame to solve the problem.

The first thing to do is calculate the speed of the ZMF. In the ZMF the particles will have speeds $u_{\mathrm{A,ZMF}} = u_{\mathrm{A}} - v_{\mathrm{ZMF}}$ and $u_{\mathrm{B,ZMF}} = u_{\mathrm{B}} - v_{\mathrm{ZMF}}$. The total momentum in the horizontal direction, which must sum to zero in the ZMF, would be given by

$$p_x = m_{\mathrm{A}} u_{\mathrm{A,ZMF}} + m_{\mathrm{B}} u_{\mathrm{B,ZMF}} = m_{\mathrm{A}}(u_{\mathrm{A}} - v_{\mathrm{ZMF}}) + m_{\mathrm{B}}(u_{\mathrm{B}} - v_{\mathrm{ZMF}}) = 0$$

Re-arranging this gives:

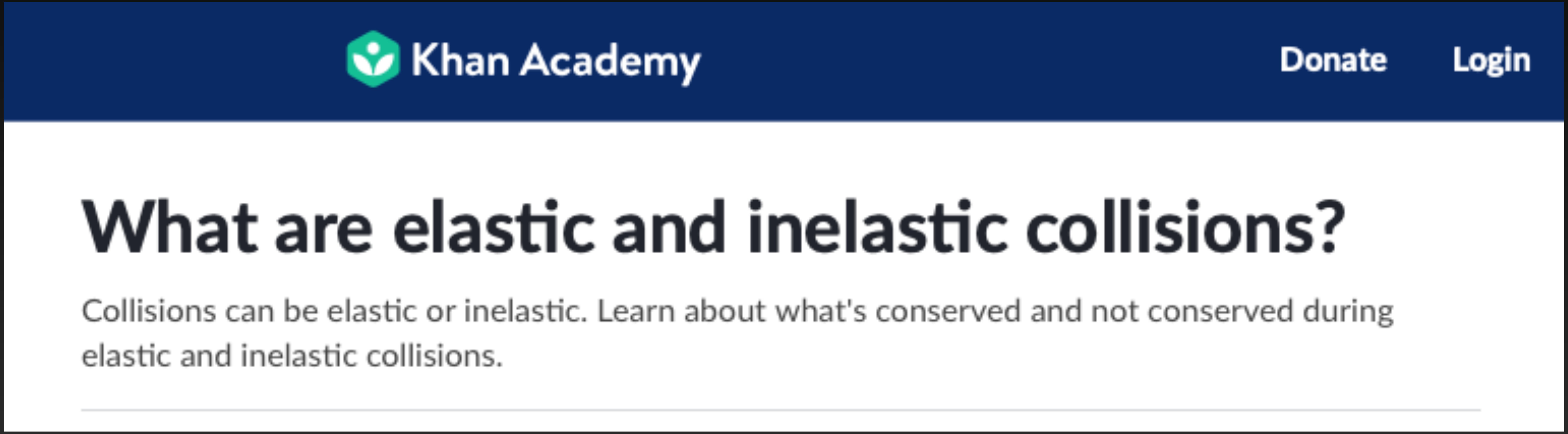
$$\begin{aligned} v_{\mathrm{ZMF}} &= \frac{m_{\mathrm{A}} u_{\mathrm{A}} + m_{\mathrm{B}} u_{\mathrm{B}}}{m_{\mathrm{A}} + m_{\mathrm{B}}} \\ &= \frac{mu}{m + 2m} \\ &= \frac{u}{3} \end{aligned}$$

so in the ZMF particle A has a speed of $u_{\mathrm{A,ZMF}} = \frac{2u}{3}$ and is moving to the right and particle B has a speed of $u_{\mathrm{B,ZMF}} = \frac{u}{3}$ and is moving to the left, as shown in the bottom left corner of Figure 7.

As the collision is elastic, both energy and momentum are conserved, and so we know that in the ZMF the particles bounce off each other with the same speeds but different directions, as shown in the bottom right hand corner of Figure 7.

To move back into the lab frame, we add $v_{\mathrm{ZMF}}$ to the velocities of each particle. This gives us a final velocity of particle A of $\underline{v}_{\mathrm{A}} = -\frac{2u}{3} + \frac{u}{3} = -\frac{u}{3}$ and the final velocity of particle B is $\underline{v}_{\mathrm{B}} = \frac{u}{3} + \frac{u}{3} = \frac{2u}{3}$.

# Additional Reference



# Key Equations

| | |
|---|---|
| Definition of momentum | $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$ |
| Impulse | $\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}(t)dt$ or $\vec{\mathbf{J}} = \vec{\mathbf{F}}_{\text{ave}}\Delta t$ |
| Impulse-momentum theorem | $\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$ |
| Average force from momentum | $\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$ |
| Instantaneous force from momentum (Newton's second law) | $\vec{\mathbf{F}}(t) = \frac{d\vec{\mathbf{p}}}{dt}$ |
| Conservation of momentum | $\frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} = 0$ or $\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{constant}$ |
| Generalized conservation of momentum | $\sum_{j=1}^{N} \vec{\mathbf{p}}_{\mathbf{j}} = \text{constant}$ |
| Conservation of momentum in two dimensions | $p_{f,x} = p_{1,i,x} + p_{2,i,x}$<br>$p_{f,y} = p_{1,i,y} + p_{2,i,y}$ |

# Activity:
# Worked Problems

**35** . Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of $1.50 \times 10^5$ kg and a velocity of $(0.30\ \text{m/s})\hat{\mathbf{i}}$, and the second having a mass of $1.10 \times 10^5$ kg and a velocity of $-(0.12\ \text{m/s})\hat{\mathbf{i}}$. What is their final velocity?

**WP 11.1**

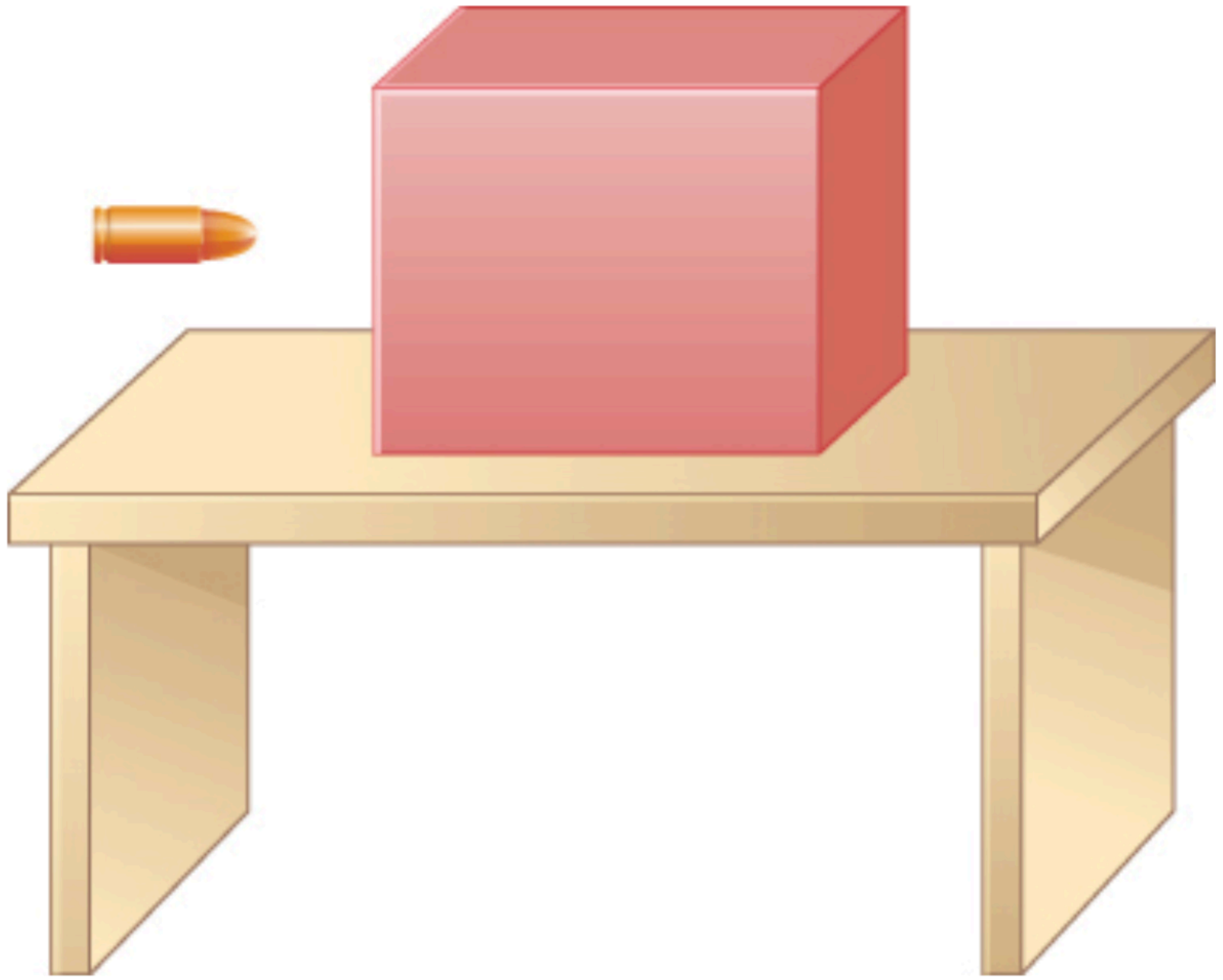
**35** . Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of $1.50 \times 10^5$ kg and a velocity of $(0.30\ \mathrm{m/s})\hat{\mathbf{i}}$, and the second having a mass of $1.10 \times 10^5$ kg and a velocity of $-(0.12\ \mathrm{m/s})\hat{\mathbf{i}}$. What is their final velocity?

**WP 11.1**


$\boxed{(0.122\ \mathrm{m/s})\hat{\mathbf{i}}}$

**37** . The figure below shows a bullet of mass 200 g traveling horizontally towards the east with speed 400 m/s, which strikes a block of mass 1.5 kg that is initially at rest on a frictionless table.

WP 11.2



After striking the block, the bullet is embedded in the block and the block and the bullet move together as one unit.

a. What is the magnitude and direction of the velocity of the block/bullet combination immediately after the impact?
b. What is the magnitude and direction of the impulse by the block on the bullet?
c. What is the magnitude and direction of the impulse from the bullet on the block?
d. If it took 3 ms for the bullet to change the speed from 400 m/s to the final speed after impact, what is the average force between the block and the bullet during this time?

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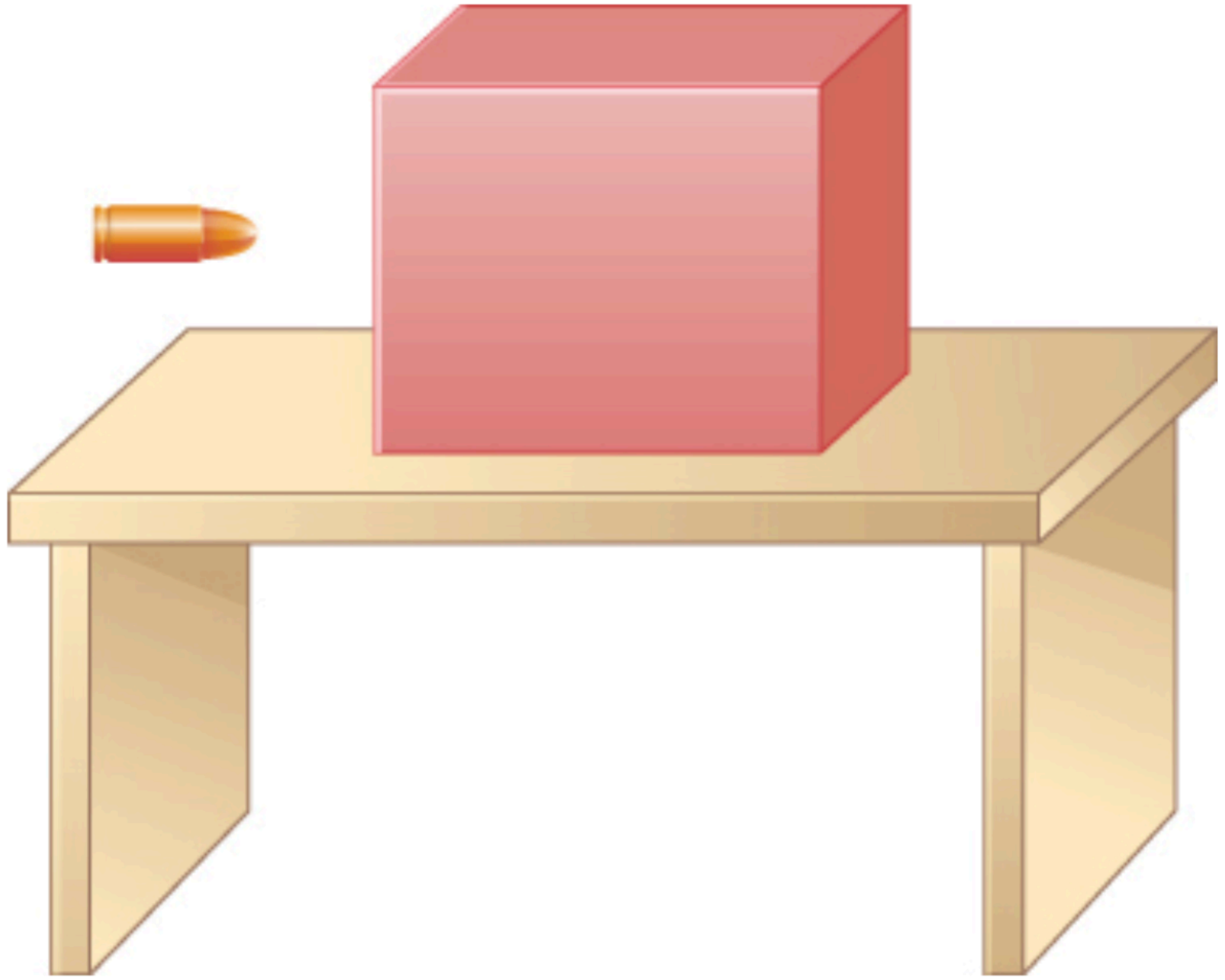

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# See you next class!

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